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PROJECT ADMINISTRATION DATA SHEET

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Title: Thermal Models for the Evolution of Magmas

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SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 3/26/85

Project No. G-35-617 School/~~Unit~~ Geo. Science

Includes Subproject No.(s) N/A

Project Director(s) Robert P. Lowell GTRC / ~~GIT~~

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Title Thermal Models for the Insulation of Magmas

Effective Completion Date: 10/31/84 (Performance) 1/31/85 (Reports)

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- [ ] None
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APPENDIX VI

NATIONAL SCIENCE FOUNDATION
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FINAL PROJECT REPORT
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PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Tech Research Corporation Georgia Institute of Technology Atlanta, GA 30332	Experimental & Theoretical Geophysics	EAR 8212717
	4. Award Period From 11/15/82 To 10/31/84	5. Cumulative Award Amount $30,000

6. Project Title

Thermal Models for the Evolution of Magmas

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

The principal objective of the research performed under this grant was to determine the conditions for the onset of convection in a growing layer of a partially molten silicate system that was heated from below. The purpose was to determine whether magmas produced by heating of a layer from below could be homogenized within the source zone. In addition to achieving the primary objective, we also were able to investigate several other important problems of a somewhat similar nature. The principal results can be summarized as follows:

a) In a growing layer of partial melt heated from below, the thickness of the layer at which convection initiates depends strongly upon the bulk composition of the silicate system. For example, in a Di-An system, finite amplitude double-diffusive convection will occur when the layer is ~800m thick if the bulk composition is Di-rich, whereas compositional convection will occur in a layer only centimeters thick if the bulk composition is An-rich. Double-diffusive processes should be included in models of melt segregation.

b) The structure of porous boundary layers in magma chambers depends upon the bulk composition. If the ratio, G, of the thermal buoyancy to the compositional buoyancy is greater than unity, the compositional boundary layer will be dragged down the wall. If G is less than unity fractionation occurs. If the viscosity is low and the diffusivity is high, fractionation rates of $\sim 10^{-3} cm^3/yr$ can occur.

c) Massive sulfide ore deposits of ~3MT or greater require that the hydrothermal system extract heat across the roof of a liquid, convecting magma chamber or extract heat by penetration of a solidifying, cooling pluton. The permeability near the top of the magma chamber can only be $10^{-15} m^2$ or less, though the solutions may exit the sea floor through highly permeable zones. Very large deposits are likely to be the result of multiple hydrothermal events.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM	
				Check (√)	Approx. Date
a. Abstracts of Theses				X	6/85
b. Publication Citations		X		X	9/85 (reprint)
c. Data on Scientific Collaborators		X			
d. Information on Inventions	X				
e. Technical Description of Project and Results		X			
f. Other *(specify)* 2 preprints 1) Double-diffusive convection ... 2) Hydrothermal models for generation		X			

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Robert P. Lowell		1/31/85

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Part III -- Technical Information

Theses

M. S. Thesis: "An analysis of double-diffusive, porous boundary layers in a magma chamber", Mr. G. Bergantz (in preparation)

Publication Citations

1) **Presentations and Papers**

a) Lowell, R. P., and J. C. Mareschal (1983). "The growth of the inner core". Spring Annual Meeting, American Geophysical Union, Baltimore, MD, May 1983 (Abstract in EOS, 64, 310, 1983).

b) Lowell, R. P., and P. A. Rona (1983). "Hydrothermal models for the generation of massive sulfide ore deposits". Annual Meeting, Geol. Soc. America, Indianapolis, IN, November 1983 (Abstract in GSA Abstracts with Programs, 15, no. 6, 552, September 1983).

c) Lowell, R. P. (1984). "Double-diffusive convection in partially molten silicate systems: Application to magma genesis". Fall Annual Meeting, American Geophysical Union, San Francisco, CA, December 1984 (Abstract in EOS, 65, 1141, 1984).

2) **Journal Articles**

a) Lowell, R. P. (1985). "Double-diffusive convection in partially molten silicate systems: Its role during magma production and in magma chambers". J. Volcanol. and Geotherm. Res. (in press).

b) Lowell, R. P., and P. A. Rona (1985). "Hydrothermal models for the generation of massive sulfide ore deposits". J. Geophys. Res. (in press).

Data on Scientific Collaborators

1) Jean-Claude Mareschal, co-investigator, Assistant Professor
2) George Bergantz, graduate student

Technical Description

The principal objective of the research performed under this grant was to determine the conditions for the onset of convection in a growing layer of a partially molten silicate system that was heated from below. The theoretical study was to parallel experimental work on the partial melting of the $Na_2Si_2O_5$ - $NaAlSiO_4$ system being concurrently performed by Dr. Hatten S. Yoder at the Carnegie Institution of Washington. The purpose of the theoretical study was to shed some light on whether magmas produced by heating of a layer from below could be homogenized in the source zone. We have achieved the principal objective of the research and have tackled some other important problems as well. We have examined the structure of double-diffusive porous boundary layers in a magma chamber and have found that appreciable fractionation rates take place only if the viscosity of the magma is low and the diffusivity of the species is high. We have also developed a model for heat extraction from the roof of a magma chamber by hydrothermal circulation and formulated a criterion for the generation of sulfide ore deposits. Finally, we developed a descriptive model of how the inner core might grow as a result of freezing at the core-mantle boundary and the subsequent precipitation of solid iron towards the center of the earth. The results of the research are summarized more fully in the following subsections. Full manuscripts of the papers in press on double-diffusive convection and on hydrothermal models for the generation of massive sulfide ore deposits are attached to this report. Reprints will be sent when they become available.

1. **Double-Diffusive Convection in Partially Molten Silicate Systems: Its Role During Magma Production and in Magma Chambers.**

In this paper, two problems involving double-diffusive convection in partially molten, silicate systems are investigated mathematically. The first problem concerns the role of double-diffusive convection during magma production. The systems Di-An and Di-An-Ab are used as examples. The calculations show that, in a growing layer of partial melt, heated from below, the thickness of the layer at which convection initiates depends strongly upon the bulk composition. For example, assuming reasonable values of the physical parameters, in a Di-rich system, convection will occur when the layer is $\sim$800 m thick, whereas in an An-rich system convection, driven by the destabilizing compositional gradient, will begin in a layer only centimeters thick. A qualitative investigation of the Di-An-Ab system suggests similar results, depending upon whether the bulk composition lies in the diopside or the plagio-clase field, respectively. Moreover, if one considers a two-component, one-dimensional diapir of Di-An, an examination of the phase diagram indicates that vigorous compositional convection will occur in the partially molten diapir as it ascends, regardless of whether the bulk composition is Di-rich or An-rich. These results suggest that during magma production, convective processes will tend to homogenize the melt before it segregates from the source zone; however, the vigor of mixing is dependent upon the bulk composition of the source, among several other factors. Models of melt segregation should be modified to include double-diffusive processes.

The second problem concerns the structure of a porous boundary layer that forms as a result of side-wall crystallization in a convecting magma chamber. An examination of the steady-state boundary layer equations for flow through a porous medium shows that the boundary layer structure may be of two types. If the residual melt fraction, upon crystallization at the wall, has negative compositional buoyancy, or if the negative thermal buoyancy at the cold wall exceeds the positive compositional buoyancy of the residual melt, the flow across the entire boundary layer will be downward. If the residual melt fraction has negative compositional buoyancy, the magma chamber will become stratified as a result of accumulation of a layer of dense, cold liquid on the floor; whereas if the melt fraction has positive compositional buoyancy, the boundary layer fluid will tend to be remixed into the interior of the magma chamber. If, however, the positive compositional buoyancy exceeds the negative thermal buoyancy, counterflowing boundary layers will occur; and the compositionally buoyant liquid will tend to be fractionated towards the top of the magma chamber. An examination of the Di-An and Di-An-Ab systems suggests that both types of boundary layer structures may occur in magma chambers. An approximate calculation of the rate of fractionation suggests that, for a range of parameters that may be representative of basaltic magmas, the flow in the compositional boundary layer may transport $\sim 10^{-3}$ km^3/yr of low density liquid to the top of the magma chamber.

2. Hydrothermal Models for the Generation of Massive Sulfide Ore Deposits.

The discovery of massive sulfide ore deposits at certain sites at active seafloor spreading centers provides a basis for considering the energetics of processes that have concentrated similar large ore deposits that have been preserved in volcanic rocks on land. In this paper we construct transport models based on hydrothermal convection systems in the oceanic crust. We investigate models in which the circulation is driven by heat extracted from the permeable crustal rocks themselves as well as by heat conducted through the roof of a replenished, vigorously convecting crustal magma chamber. If the hydrothermal fluids carry 100 ppm dissolved iron as found in black smokers on the East Pacific Rise and exit at 350^{o}C or greater, calculations show that the heat content of the oceanic crust is insufficient to account for ore deposits of 3MT or greater. Heat extraction through the roof of a magma chamber, in conjunction with the formation of a thin ($\sim$1 km) layer of plated gabbro, may account for ore bodies of the order of 3MT, provided the permeability of the oceanic crust in the magma-hydrothermal fluid contact zone is 10^{-15} m^2 or less. If the conducting boundary layer were only one meter thick, locally, for 10^3 years, the permeability could be as high as 10^{-13} m^2. The permeability in the discharge zone could, conceivably, be two orders of magnitude greater than the permeability at depth. More work is needed on the details of the effects of magma chamber replenishment and off-axis dike injection and on the role of magmatic fluids. Very large ore deposits (i.e., $\sim$100MT) are probably formed in multi-stage events.

3. The Growth of the Inner Core.

We considered the effects of freezing at the core-mantle boundary (CMB) on the evolution of the earth's outer and inner cores. We assumed the composition of the outer core to be a metal-rich, eutectic-like system. We argued that if freezing takes place at the CMB, there will exist thermo-chemical boundary layers ot the top of the core and just above the inner core - outer core boundary (OCB). We suggested that the inner core could grow as a result of direct precipitation of iron frozen out in the boundary layer near the CMB, provided it did not remelt upon descent through the core. If the iron did remelt, the outer core could become gravitationally unstable, and after gravitational overturn, the iron could then refreeze out of the metal-rich solution that would then lie towards the center of the earth. Freezing at the CMB may occur in conjunction with the formation of convective plumes in the deep mantle.

DOUBLE-DIFFUSIVE CONVECTION IN PARTIALLY MOLTEN SILICATE SYSTEMS: ITS ROLE DURING MAGMA PRODUCTION AND IN MAGMA CHAMBERS

By

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J. Volcanology and Geothermal Research

December 1984

ABSTRACT

In this paper, two problems involving double-diffusive convection in partially molten, silicate systems are investigated mathematically. The first problem concerns the role of double-diffusive convection during magma production. The systems Di-An and Di-An-Ab are used as examples. The calculations show that, in a growing layer of partial melt, heated from below, the thickness of the layer at which convection initiates depends strongly upon the bulk composition. For example, assuming reasonable values of the physical parameters, in a Di-rich system, convection will occur when the layer is ~800 m thick, whereas in an An-rich system convection, driven by the destabilizing compositional gradient, will begin in a layer only centimeters thick. A qualitative investigation of the Di-An-Ab system suggests similar results, depending upon whether the bulk composition lies in the diopside or the plagioclase field, respectively. Moreover, if one considers a two-component, one-dimensional diapir of Di-An, an examination of the phase diagram indicates that vigorous compositional convection will occur in the partially molten diapir as it ascends, regardless of whether the bulk composition is Di-rich or An-rich. These results suggest that during magma production, convective processes will tend to homogenize the melt before it segregates from the source zone; however, the vigor of mixing is dependent upon the bulk composition of the source, among several other factors. Models of melt segregation should be modified to include double-diffusive processes.

The second problem concerns the structure of a porous boundary layer that forms as a result of side-wall crystallization in a convecting magma chamber. An examination of the steady-state boundary layer equations for flow through a porous medium shows that the boundary layer structure may be of two types. If the residual melt fraction, upon crystallization at the wall, has negative compositional buoyancy, or if the negative thermal buoyancy at the cold wall exceeds the positive compositional buoyancy of the residual melt, the flow across the entire boundary layer will be downward. If the residual melt fraction has negative compositional buoyancy, the magma chamber will become stratified as a result of accumulation of a layer of dense, cold liquid on the floor; whereas if the melt fraction has positive compositional buoyancy, the boundary layer fluid will tend to be remixed into the interior of the magma chamber. If, however, the positive compositional buoyancy exceeds the negative thermal buoyancy, counterflowing boundary layers will occur; and the compositionally buoyant liquid will tend to be fractionated towards the top of the magma chamber. An examination of the Di-An and Di-An-Ab systems suggests that both types of boundary layer structures may occur in magma chambers. An approximate calculation of the rate of fractionation suggests that, for a range of parameters that may be representative of basaltic magmas, the flow in the compositional boundary layer may transport $\sim 10^{-3}$ km^3/yr of low density liquid to the top of the magma chamber.

INTRODUCTION

Double-diffusive convection is a type of instability that may occur when a compositional as well as a thermal gradient occurs in a fluid. The instability results from the fact that the thermal diffusivity is generally several times (perhaps even many orders of magnitude) greater than the mass diffusivity. If, for example, the thermal gradient is destabilizing and the compositional gradient is stabilizing, a parcel of fluid displaced vertically upward from its equilibrium position will lose heat faster than solute. The resulting bouyancy force will tend to reverse the displacement of the parcel and an oscillating instability ensues. Veronis (1965) gives a more thorough introductory discussion of this process and shows that under certain circumstances a finite amplitude monotonic instability may set in under a less restrictive condition than the oscillatory instability. Huppert and Turner (1981) provide an up-to-date review of the subject.

Double-diffusive convection was first recognized as a potentially important process in the ocean, where opposing temperature and salinity gradients give rise to the instability (see Turner, 1973). In recent years, however, the observation that compositional gradients are likely to occur in magma chambers in the earth's crust has led to application of the double-diffusive concepts in the investigation of the dynamics of these systems as well. McBirney and Noyes (1979) have applied this concept to explain the layering in the Skaergaard intrusion; and Irvine (1980) points to the possible importance of the double-diffusion in the crystallization of the Muskox complex. Hildreth (1981) argues that

double-diffusive processes give rise to liquid state fractionation in rhyolitic magmas; the stratification in the magma chamber is then preserved, in a somewhat inverted order, in the extensive rock sequences. These field observations have provided the impetus for the continuing series of laboratory investigations by Turner and his coworkers (e.g., Chen and Turner, 1980; Turner and Gustafson, 1981; Kerr and Turner, 1982; Huppert et al., 1983) and McBirney (1980), McBirney et al. (1984).

Despite the abundance of recent work on double-diffusive convection in magma chambers, there has been a notable sparcity of discussion of the effect of crystals on the dynamics of the convection systems. The theoretical investigations to date have nearly all dealt with the dynamics of crystal-free viscous fluids. It seems probable, however, that when crystallization occurs near the boundaries of a multi-component magma chamber, the crystals will not form an impermeable crystal lattice that will act as a smooth wall. Rather the crystal structure will tend to be somewhat open. Fluids occupying the interstitial spaces may react chemically with the crystal matrix. In the event of external driving forces, e.g., thermal and/or compositional bouyancy, the fluids may flow through the crystal matrix in a manner analogous to Darcian flow through a porous medium. The permeability of the crystalline matrix will vary spatially as will the viscosity of the interstitial melt. The nature of the flow in a crystallizing, permeable boundary layer in a multi-component magma chamber is of considerable importance in determining the thermal and chemical evolution of the magma.

A porous medium flow model might also be a suitable approximation for the dynamics of partial melts formed during magma genesis (Turcotte and Ahern, 1978; Sleep, 1974; Yoder, 1978), since the melt products tend to concentrate along interconnected grain boundaries, thereby providing permeability (Arndt, 1977). A description of the behavior of the melt during magma production in terms of a porous flow model may be complicated by factors such as (a) the possibility that a critical melt fraction must be attained before permeability is established (Waff, 1980), and (b) the tendency for the melt fraction to segregate from the crystalline matrix and form pools of liquid (Walker et al, 1978; Stolper et al., 1981; McKenzie, 1984). Nevertheless, it is noteworthy that none of the treatments to date of a partially molten magma source region has considered the possible effects of temperature and compositional gradients that are likely to occur across the partially molten region. Such gradients may have an important effect on the dynamics of the source zone and the magma transport mechanism, and may also be a factor in determining the chemical signature of melts that rise towards the earth's surface.

In this paper two problems of double-diffusive convection in a partially molten silicate system will be investigated. The first problem will be concerned with double-diffusive effects that may arise in a partially molten zone that occurs during magma generation. The partially molten zone will be assumed to result from either the heating of a horizontal layer of silicate rock from below or from the ascent of a diapir into a pressure-temperature regime where partial melting takes

place. The second problem will be concerned with the possible occurrence of counterbuoyant, porous-flow boundary layers in a magma chamber. In order to remove some of the complexities that exist in real, multicomponent silicate systems, idealized systems will be considered. In particular, the binary, eutectic-like, diopside-anorthite system, and the ternary system, diopside-plagioclase, will be used. In the present analysis the idealized situation, in which a sufficient melt fraction is present to result in a permeable, crystalline matrix, will be considered. Moreover, it will be assumed that, on the time scales considered, the crystalline matrix does not deform and that melt segregation can be neglected.

BASIC EQUATIONS

The basic equations needed to describe the flow of an incompressible two-component fluid through a rigid, permeable matrix are the conservation of fluid mass, fluid momentum, energy, and the individual chemical components. These equations are, respectively:

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

$$(\rho_\ell/\chi)\ \partial u/\partial t = -\nabla P - (\eta/K)\vec{u} - \rho_\ell \vec{g} \tag{2}$$

$$\rho c_p\ \partial T/\partial t + \rho_\ell c_\ell \vec{u} \cdot \nabla T = \kappa \nabla^2 T \tag{3}$$

$$\partial C/\partial t + \vec{u} \cdot \nabla C = D\nabla^2 C + J \tag{4}$$

The symbols are defined in the NOMENCLATURE. In addition to the conservation equations, one requires an equation of state and a phase diagram for the system. The equation of state will be assumed to be of the form

$$\rho_\ell = \rho_{\ell o}\ (1-\alpha(T-\overline{T}) - \beta(C-\overline{C})) \tag{5}$$

where the barred quantities are reference values of the respective variables. If one is considering processes that are occurring at pressures of tens of kilobars, or are taking place over a considerable range of pressure, it may be important to include the effects of compressibility of the melt fraction (Stolper et al., 1981). The phase diagrams for the systems discussed below are given in Figures 1 through 4.

DOUBLE-DIFFUSIVE CONVECTION DURING MAGMA PRODUCTION

The process or processes by which magma is generated in the earth's crust and mantle is not well understood. Yoder (1976) reviews several possible mechanisms for the generation of basaltic magma. Some of the proposed mechanisms may be applicable to the generation of more silicic magmas as well. Among the mechanisms are heating of a layer of material from below and diapiric ascent. These mechanisms, though somewhat simplistic, are, nevertheless, instructive for the purpose of investigating the possible role of double-diffusive convection during magma generation. Using the phase diagram for Fo-Di-Py (Davis and Shairer, 1965) as an example, Yoder (1978) showed that the heating of a layer of garnet-peridotite from below would give rise to vertical gradients in the temperature, the percent of partial melt, and the melt composition in a partially molten layer that would grow with time. Similarly, he showed that a garnet-peridotite diapir ascending adiabatically could become partially molten and that the melt fraction and its chemical composition would tend to vary as a function of depth within the partially molten region. For the system Yoder chose, the compositional gradient in the partially molten diapir would tend to be opposite to that for a partially molten layer heated from below. One of the points of Yoder's discussion is that magmas generated in the presence of temperature and pressure gradients tend to exhibit some degree of chemical heterogeneity and that, in order to explain the large volumes of relatively homogeneous magmas erupted at the earth's surface at particular locations over a considerable period of time (e.g., the Columbia River

basalts), the magmas must be homogenized either within the source zone or within subsidiary, shallow magma chambers.

A. A Layer Heated From Below

To investigate the possibility of homogenization of initially compositionally-layered melt within the source zone, consider first the condition for which double-diffusive convection will occur in a growing, partially molten layer, heated from below. As examples, materials with bulk compositions labelled a, b, c, d in Figures 1 and 3, respectively, will be considered. In particular, suppose a layer of material composed of diopside and anorthite and having the diopside-rich, bulk composition a, has somehow been heated so that the entire layer is at the eutectic temperature T_e, but that no melting has occurred. Then suppose that at time $t = 0$, the base of the layer, $z = 0$, is raised to a temperature $T^* > T_e$, as a result of the injection of a sill, for example. Then for $t > 0$, as latent heat is absorbed, the partial melt front will move upward through the material. At time $t = t_1$, say, the front will be $z = h(t_1)$. The region $0 < z < h(t_1)$ is partially molten, and there will be a temperature and a compositional gradient across the region. Figure 5 shows the configuration at time t.

To solve the mathematical problem governing the growth of the partially molten layer and movement of this melting front is quite difficult, because the slow rate of mass diffusion inhibits phase boundary motion and transient concentration gradients will develop as the front advances. For purposes of mathematical simplicity, it will be

assumed that the melt boundary motion is controlled by thermal diffusion only (i.e., the classical Stefan problem) and that the chemical composition of the liquid at any point in the partially molten layer is given as a function of temperature by the equilibrium phase diagram. With this assumption, it is clear from the phase diagram in Figure 1 that the melt will be enriched in diopside towards the base of the layer. The Lever Rule, as applied to starting composition a, shows that the melt fraction also increases towards the base. The melt fraction is identified with the porosity in the porous medium model. Since the physical properties of the melt and crystal matrix are also dependent upon porosity (e.g., thermal conductivity, permeability, etc.), it is apparent that one is still faced with a highly complex mathematical problem.

The mathematical problem of convection in a growing, partially molten layer, heated from below, is analysed in two stages. The first stage consists of a purely thermal problem, in which the effects of latent heat and the motion of the phase boundary are included in the convective stability analysis. The chemical composition of the melt is assumed to be constant across the layer, as are the other physical parameters. This problem is analogous to that of Sparrow et al. (1976), who derived the condition for the onset of convection in a water layer that formed beneath a layer of ice heated from below. This formulation of the problem extends the rudimentary analysis of Lowell (1982). Because of the effects of the latent heat and phase boundary motion on the critical condition for the onset of convection turn out to be small, the detailed mathematical analysis is relegated to the Appendix. The

result derived there is that the Rayleigh number, $R = \alpha g K \Delta T h/\lambda\nu$ must exceed a critical value, R_c, where

$$R_c \gtrsim 4\pi^2(1-Q^2/2\pi^2) \qquad (6)$$

where Q is a dimensionless parameter, assumed to be less than unity. The standard stability criterion is $R_c \geq 4\pi^2$ (Lapwood, 1948). Q is less than unity provided the Stefan number is less than 2.29. If the latent heat required to melt 20% of the rock layer is taken to be 50 cal/gm, and the specific heat of the partially molten layer is 0.2 cal/gm-^oC, the result (6) is valid so long as the temperature difference across the layer is less than 570^oC. Thus the effect of latent heat and phase boundary motion on convective instability is negligible over the range of temperature differences that might normally be encountered in magmatic systems. The second stage of the analysis involves a semi-quantitative treatment of the effect of the compositional gradient on the stability of the partially molten layer. The results of the first stage of the analysis indicate that it is safe to neglect the dynamics of the melt front and the effects of latent heat when evaluating double-diffusive effects.

1. Basic Criteria for the Onset of Double-Diffusive Convection

The stability problem for double-diffusive convection in a porous medium has received comparatively little attention compared to the analogous problem for a viscous fluid. The basic paper is that of Nield (1968) who showed that, in a layer of thickness h on which the physical properties are uniform and across which uniform thermal and

concentration gradients were maintained, the stability was controlled by two dimensionless parameters. They are the ordinary thermal Rayleigh number given previously and a concentration Rayleigh number $R^* = \beta g \Delta c K h / D\nu$. Nield showed that, for rigid boundaries, monotonic instability to infinitesimal perturbations sets in when

$$R + R^* \geq 4\pi^2 \tag{7}$$

and that an oscillatory instability sets in when

$$R(1 + Le^{-1}) + R^*(1 + Le)^{-1} \geq 4\pi^2 \tag{8}$$

where $Le = \lambda/D$ is the Lewis number. Since $Le \gg 1$ for silicate liquids, equation (8) reduces to

$$R + R^*/Le \geq 4\pi^2 \tag{9}$$

Recently, Rudaiah et al. (1982) and Lowell (unpublished calculations), following Veronis (1965), showed that if the concentration gradient were stabilizing, a finite amplitude, monotonic instability could occur if:

$$R^f \geq (2\pi(1-Le^{-2})^{1/2} + Le^{-1}(-R^*)^{1/2})^2 \tag{10}$$

In the notation used here, a stabilizing concentration gradient implies $R^* < 0$. Thus if $|R^*|/Le^2 \gg 4\pi^2$, and $Le \gg 1$, then the finite amplitude instability occurs when

$$R^f \geq Le^{-2} \, |R^*| \tag{11}$$

whereas the infinitesimal, oscillatory instability occurs when

$$R \geq Le^{-1} |R^*| \tag{12}$$

Equations (11) and (12) mean that a monotonic finite-amplitude instability can occur even if the overall density gradient is gravitationally stable, but that the oscillatory instability will occur only if the overall density gradient is gravitationally unstable. If $|R^*|/Le^2 \ll 4\pi^2$, the finite-amplitude instability will occur when $R^f \geq 4\pi^2$ and this instability will still occur first. Equation (7) shows that for monotonic, infinitesimal instability to occur, the density gradient must be highly unstable. It is also important to realize that if the compositional gradient is destabilizing, as can be the case for some silicate liquids composition that may occur during magma production. Then ordinary, Lapwood-type convection may occur in a very thin layer of partial melt.

2. Application to Partially Molten Silicate Systems--Some Examples

The critical Rayleigh numbers given in the previous section for the various modes of instability, corresponding to Nield's (1968) model, do not exactly correspond to the model of a growing, partially molten layer, heated from below. The principal difference is that in Nield's model the compositional gradient was held constant, implying that a source and sink of solute were maintained below and above the layer, respectively. In the model envisioned in this paper, the upper and lower boundaries are impervious to diffusion of chemical components. This difference in boundary conditions will tend to lower the numerical

value required of the critical parameters, perhaps by a factor of 4 or so (see Table 1 of Nield (1968)), but will not change the main conclusions to be derived below. The assumptions made herein that the physical parameters are constant (e.g., permeability, melt fraction, viscosity, etc.) are also not thought to be critical to the present analysis.

Consider first the Di-An eutectic system with bulk composition a (i.e., 95% Di). Let the temperature at the base of the layer be 1350°C and at the top 1274°C, so that the melt fraction is everywhere small enough so that crystal bridging is maintained and the porous medium model holds. From the liquid densities in Figure 1, calculated on the basis of Mo et al. (1982), it is observed that the compositional gradient is stabilizing, but that the overall density gradient is destabilizing. The ratio of the thermal buoyancy to the compositional buoyancy $G = \alpha\Delta T/\beta\Delta C \cong 1.48$. Oscillatory instability is possible, therefore, but the finite amplitude instability may occur at a smaller value of R. Since the compositional gradient is only slightly stabilizing and since the Lewis number probably ranges from 10^3 to 10^7 (see Table 1 for numerical values of parameters used in this paper), $R^*/Le^2 \ll 1$. Thus the finite amplitude, double-diffusive instability will occur when the height of the partially molten layer is given by

$$h \geq \frac{4\pi^2 \lambda \nu}{\alpha \Delta T K g} \tag{13}$$

The result (13) is formally identical to the result of Lowell (1982), which was based on the assumption that the composition was

constant across the layer. The nature of the instability is different in the two cases, however. Letting $K = \chi^2 b^2/72\pi$ (Ahern and Turcotte, 1979; Lowell, 1982), with $\chi = 0.1$, $b = 10^{-1/2}$ cm, the other parameters taken from Table 1, and with $\alpha\Delta T \cong 1.12 \times 10^{-2}$ from Figure 1, composition a, equation (13) gives $h \geq 800$ m, which is not an unreasonable value. Such a layer could form by heating from below in approximately 10^4 years (see Appendix, equation (A6)), and melt segregation is unlikely to be a problem (Stolper et al., 1981).

Consider next the composition b (90% An) in Figure 1. The striking feature is that the melt at the base of the layer is enriched in the An component relative to the top. The compositional gradient is destabilizing. Hence monotonic, Lapwood-type convection will occur, driven by compositional rather than thermal buoyancy. The layer thickness at which convection begins can be estimated from equation (7):

$$h \geq \frac{4\pi^2 D\nu}{\beta\Delta C K g} \tag{14}$$

From the phase diagram, Figure 1, $\beta\Delta C \cong 7.6 \times 10^{-3}$. Taking the same values of parameters as in the previous calculation gives h in the range from 10^{-2} cm to 10^2 cm for D in the range 10^{-9} to 10^{-5} cm^2/s, respectively. In the lower end of the range, the height of the convecting layer is considerably less than the assumed grain size of the crystalline matrix. The porous medium model breaks down. Compositional convection at these length scales may, in fact, occur during crystal growth (Coriell et al., 1980; see also Huppert and Sparks, 1984), but the

dynamics are not governed by the laws of flow in a porous medium. In any case, compositional convection will occur in a very thin layer, provided both the compositional and thermal gradients are destabilizing.

Results similar to those for cases a and b can be obtained from a consideration of the ternary system, Di-An-Ab, shown in Figure 3. Consider first the diopside-rich field depicted, for example, by bulk composition c. Melt compositions that form by equilibrium melting in a destabilizing temperature gradient have a density that increases with increasing temperature. The density distribution is gravitationally stable. Finite amplitude, double-diffusive convection will occur at layer height similar to that given by equation (13). In this case, because the compositional gradient is relatively more stabilizing, the critical height may be a little greater than in the diopside-rich binary system considered previously. A somewhat more interesting situation is presented by consideration of a bulk composition in the plagioclase field, e.g., d. In this case, the first liquids formed by equilibrium melting in a destabilizing temperature gradient lie on the cotectic, and hence the density distribution is stable as in c. If the temperature at the base of the heated layer is high enough, however, the melt composition near the base will be more plagioclase-rich, and of lower density, than the overlying melt. The density distribution in the melt fraction will undergo a density inversion and mixing will take place. Because the compositional gradient is not destabilizing across the entire layer, the dynamics in this case will be somewhat different than in case b. The details of the convective instability in a situation

like that presented by case d have yet to be worked out. Naturally occurring partially molten silicate systems are, of course, considerably more complex than the idealized systems considered here; and because the density variations in the melt layer may depend upon the bulk composition and the degree of partial melting, among other factors, exact nature of the convective instabilty (whether double-diffusion or compositional) will be difficult to predict a priori. It does appear that either process might occur in a relatively thin layer of partial melt, thereby giving rise to the possibility that the melt may be homogenized before it leaves the source zone.

B. A Rising Diapir and Solidifying Plutons

The problem of heat transfer associated with an ascending diapir is quite complex and will not be treated here. Marsh and Kartha (1978), Turcotte and Ahern (1978), and Ribe (1984) illustrate some aspects of the problem. In the present context, the point to be made follows from the suggestion of Yoder (1978) that the pressure-release partial melting of an appropriate bulk composition in the system Fo-Di-Py will tend to result in a vertical compositional gradient in the diapir. A similar compositional gradient would also occur in the binary system Di-An. As shown in Figure 2, the effect of increasing pressure is to shift the eutectic towards An and to raise the eutectic temperature. If one considers as a simple example, a one-dimensional diapir (i.e., no lateral heat transfer), with the bulk composition on either the diopside-rich or anorthite-rich side of the eutectic, and thermodynamic equilibrium over the melting range, the composition of the melt within

the diapir will be enriched in An towards the bottom. Vigorous compositional convection will occur within the diapir as it ascends. Similarly, if one considers the ternary system shown in Figure 4, whether double-diffusive or compositional convection will occur will depend upon the composition of the first liquid formed on the cotectic as a function of pressure. The effect of pressure on the melt density has been neglected in both of these qualitative examples. Batiza and Vanko (1984) point out that there are significant petrological differences among seamount basalts and suggest that these differences stem from different degrees of partial melting and varying amounts of convective mixing during melt segregation.

Finally, it is worth pointing out that the discussion on convection in a growing layer of partially molten rock can be turned around so as to apply to the situation of a freezing layer of partially molten rock. Crystals at the base of a cooling intrusion may form an interconnected permeable matrix fluid with melt. As cooling, and crystallization, continues, the density of the residual fluid may either decrease or increase, depending upon the bulk composition, phase relationships, etc. If the density of the residual fluid increases as the rock is cooled from above, compositional convection may occur in parts of the intrusion until it is virtually completely solidified. This may be the case for the Skaergaard intrusion (McBirney and Noyes, 1979).

DOUBLE DIFFUSIVE POROUS BOUNDARY LAYERS IN A MAGMA CHAMBER

It is generally accepted that magma chambers undergo vigorous convection during some part of their lifetime. It is to be expected, therefore, that near the boundaries much of the heat transfer takes place through narrow boundary layers. The nature of the boundary layers has received comparatively little theoretical analysis. Spera et al. (1982) have considered the effects of large viscosity changes across the boundary layer in a viscous fluid, but they neglect double-diffusion. Nilson and Baer (1982) derive conditions for the occurrence of counter-buoyant, double-diffusive boundary layers in a viscous fluid, but they do not apply their results directly to magma chambers. They neglect the effects of latent heat and crystallization as well as the effect of temperature dependent viscosity. Nilson et al. (1984) and Spera et al. (1984) have included the effect of varying viscosity and have attempted to apply their results to magma chambers.

Chen and Turner (1980), Huppert et al. (1983), Turner and Gustafson (1981), McBirney (1980), and McBirney et al. (1984) have performed laboratory experiments that relate double-diffusion to processes in magma chambers. In particular, McBirney (1980) and Turner and Gustafson (1981) show the occurrence of streams of low density fluid that rise from the crystals that grow along the walls of the container. Moreover, the laboratory experiments of Chen and Turner (1980) and McBirney (1980) show that a complex network of crystals forms at the margins of the container. Apparently, the magma chamber "wall" is not flat and the structure of the flow in the boundary layer may be quite complicated.

The Nilson and Baer (1982) and Nilson et al. (1984) models of double-diffusive boundary layers along a flat vertical wall may be inappropriate. A better first approximation to the boundary layer structure might come from treating the boundary layer region as a porous medium. The principal purpose of this paper is to derive a condition for the occurrence of counterbuoyant, double-diffusive boundary layers in a porous medium.

A. The Boundary Layer Equations

In the boundary layer, the diffusion of heat and concentration perpendicular to the boundary are much greater than parallel to it. A schematic of the assumed two-dimensional mdoel magma chamber is shown in Figure 6. Following the analysis of Cheng and Minkcowycz (1977), the steady-state boundary layer equations can be taken from equations (1) through (5).

$$\nabla \cdot \vec{u} = 0 \tag{15}$$

$$-\nabla P - (\eta/K)\,\vec{u} - \rho_\ell'\vec{g} = 0 \tag{16}$$

$$\vec{u} \cdot \nabla T = \lambda \partial^2 T/\partial z^2 \tag{17}$$

$$\vec{u} \cdot \nabla C = D \partial^2 C/\partial z^2 \tag{18}$$

$$\rho_\ell' = -\rho_o \alpha (T-T_\infty) - \rho_o \beta (C-C_\infty) \tag{19}$$

where $\lambda = \kappa/\rho_\ell C_\ell$, and the subscript ∞ refers to the values of temperature and concentration in the interior of the fluid.

To obtain a similarity solution, a dimensionless similarity variable $\mu = C^*(y/z^{1/2})$ is introduced, where C^* is a constant. Further, let $\theta = (T_\infty - T)/(T_\infty - T_w)$ be a dimensionless temperature and $\phi = (C-C_\infty)/C_w-C_\infty)$ be a dimensionless concentration, where the subscript w refers to values of T and C at the wall. Finally, the vertical velocity $w = 2\nu_\infty C^{*2} f'(\mu)$, where ν_∞ is the reference kinematic viscosity far from the wall of the magma chamber. From the continuity equation (15), one obtains the horizontal velocity

$$v = \nu_\infty C^* z^{-1/2}[\mu f'(\mu) - f(\mu)] \tag{20}$$

and upon substituting for w and v into the other equations, one obtains

$$\theta'' + Pr f\theta' = 0 \tag{21}$$

$$\phi'' + Scf\phi' = 0 \tag{22}$$

where $Pr = \nu_\infty/\lambda$ and $Sc = \nu_\infty/D$ are the Prandtl and Schmidt numbers, respectively. The vertical momentum equation, assuming that there is a balance between viscous and buoyancy forces, is

$$2\nu_\infty C^{*2} f'(\mu) = g\left(\frac{K}{\nu}\right)_\infty \zeta(\theta,\phi)[-\alpha(T_\infty - T_w)\theta + \beta(C_w - C_\infty)\phi] \tag{23}$$

where the parameter $\zeta(\theta,\phi)$ represents the possible temperature and compositional dependence of the ratio K/ν, the permeability divided by the kinematic viscosity. Upon substituting $C^{*2} = (K/2\nu^2)_\infty\ g[\alpha(T_\infty - T_w) + \beta(C_w - C_\infty)]$ into (23), that equation becomes

$$f'(\mu) = \zeta(\theta,\phi)\left[-\frac{G}{1+G}\theta + \frac{1}{1+G}\phi\right] \tag{24}$$

where $G = \alpha(T_\infty - T_w)/\beta(C_w - C_\infty)$ and is positive. Equations (21), (22), and (24) are the coupled, ordinary differential equations for dimensionless temperature, concentration, and vertical velocity. The boundary conditions are:

$$\theta(0) = \phi(0) = 1 \tag{25}$$

$$\theta(\infty) = \phi(\infty) = 0 \tag{26}$$

$$f(0) = f'(\infty) = 0 \tag{27}$$

B. Boundary Layer Regimes

The principal aim of this paper is to investigate the existence of counterflowing boundary layers. This can be done even without solving the coupled set of boundary layer equations. For simplicity, let $\zeta(\theta,\phi) = 1$ (i.e., the viscosity and permeability are assumed constant) in (24). The assumption of constant parameters would appear to be a serious oversimplification of the real state of affairs in a magma chamber. Nilson et al. (1984), however, show that the effects of variable viscosity on the structure of the boundary layers in a viscous fluid bounded by a flat vertical wall can be accounted for by using a suitably chosen mean viscosity. The thermal boundary layer thickness $\delta_T \sim \ell/R_b^{1/2}$, and the compositional boundary thickness $\delta_c \sim \ell/R_b^{*1/2}$, where ℓ is a scale height and R_b and R_b^* are the thermal and compositional Rayleigh numbers based on ℓ and on the temperature and compositional differences across the boundary layer, respectively. The ratio

$\delta_T/\delta_C \sim (Le/G)^{1/2}$. Therefore, $\delta_T/\delta_C >> 1$ unless G is considerably larger than might be expected in magma chambers.

Following Nilson and Baer (1982), if it is assumed that the compositional and thermal buoyancy forces act in opposite directions, one can consider three flow regimes: inner dominated flow in which the boundary layer motion is controlled by the inner, or compositional, boundary layer; outer dominated flow in which the boundary layer motion is controlled by the outer, or thermal, boundary layer; and counterflow in which the compositional boundary layer flow is upward and the thermal boundary layer flow is downward as depicted in Figure 6. It is possible, of course, that in real magmas, depending upon the bulk composition, phase relationships, etc., the compositional and thermal buoyancy forces are both directed downward in the boundary layer. There will then be fractionation of a dense liquid towards the bottom of the magma chamber rather than fractionation of a light liquid towards the top of the chamber.

Inner dominated flow can occur, in principle, when positive compositional buoyancy is considerably greater than negative thermal buoyancy. If $G << 1$ in (14):

$$f' \cong -G\theta + \phi \tag{28}$$

If $G\theta < \phi$ for all μ, $f' > 0$ everywhere and the flow is upward in both the compositional and thermal boundary layers. Outer dominated flow can occur when negative thermal buoyancy is considerably greater than positive compositional buoyancy. Assuming $G >> 1$ in (24) gives:

$$f'(\mu) \cong -\theta + (1/G)\phi \tag{29}$$

If $\theta > (1/G)\phi$ for all μ, $f' < 0$ everywhere. In fact, one can see by direct inspection of (24), the boundary conditions (25), (26), and the fact that $\delta_T/\delta_c >> 1$ for reasonable values of Le and G, that so long as $G > 1$, the flow is outer dominated. At the wall (25) shows that $\theta = \phi = 1$, but because of the relative thickness of the boundary layers, ϕ decreases to zero faster than θ. Therefore, if G is greater than unity the first term on the right-hand side of (24) is always greater than the second term and $f'(\mu) < 0$ everywhere. On the other hand, if G is less than unity, (25) and (24) require that $f'(\mu) > 0$. The flow is upward at the wall. At a short distance, say δ_c, from the wall, however, $\theta \sim 1$, $\phi \sim 0$ so that even if G is considerably less than unity, there will be a region beyond δ_c for which $f'(\mu) < 0$. Thus counterflow occurs. Since the upward buoyancy is confined to a narrower plume as G decreases it would appear that inner dominated porous boundary layer flow is unlikely to occur in a magma chamber. Since the parameter $\zeta(\theta,\phi)$ which represents the variations in permeability and kinematic viscosity acts merely as a multiplicative factor in (24), the results concerning the occurrence of counterflowing vs. outer dominated or inner dominated boundary layers are independent of these variations. It is important to note that the criteria for the different boundary layer regimes in the porous medium are different from those in a viscous fluid. The difference stems primarily from the simpler viscous stress relationship in the porous medium case and because the vertical velocity in the porous boundary layer is not required to vanish at the wall.

C. Sample Calculations

The preceding results may be applied to binary silicate partial melts in which the temperature in the boundary layer is below the liquidus. For example, consider again the bulk compositions a and b in the Di-An system (Figure 1).

If one considers case a, Di 95% mixture, the liquidus temperature is ~1375^{o}C and the eutectic is 1274^{o}C. Assuming thermodynamic equilibrium, the anorthite component of the melt decreases with distance from the wall of the magma chamber. The compositional buoyancy is positive, but the thermal buoyancy is greater than the compositional buoyancy. Since G is greater than unity the boundary layer is outer dominated and the flow is directed downward all along the wall. The anorthite-rich fluid, upon arriving at the bottom of the magma chamber, should tend to remix with the bulk of the fluid in the magma chamber.

Consider next case b, An 90%. In this case the anorthite component in the melt increases with distance from the wall of the magma chamber. Compositional as well as thermal buoyancy is negative. The flow is directed downward across the entire boundary layer, and the fluid in the boundary layer, because of its composition and density, should be fractionated towards the base of the magma chamber.

Finally, consider cases c and d in the ternary system shown in Figure 3. In case c, the compositional buoyancy is directed upwards and exceeds the thermal buoyancy. Case c would, therefore, provide an example of counterflowing boundary layers in a magma chamber. Case d

is somewhat more complicated. So long as the melt composition is on the cotectic the compositional buoyancy is directed upward as in case c. If the melt composition moves from the cotectic into the plagioclase field, however, compositional buoyancy changes direction. The effect that this has on the boundary layer dynamics may depend on the steepness of the thermal gradient between the wall and the interior. Further analysis is unwarranted in this paper; however, the dynamics of boundary layers in more complicated systems such as this might be a worthy topic for laboratory investigation.

D. Fractionation Due to Counterflowing in a Porous Boundary Layer

The preceding semi-quantitative analysis suggests that the dynamics of the boundary layers in magma chambers are a strong function of the bulk composition and phase relationships, among other factors. It is quite conceivable that the motion in the boundary layer could change with time as certain phases are removed by crystallization. It appears to be likely, however, that in some cases, upward compositional buoyancy may result in fractionation of a low density melt towards the top of the magma chamber. A rough estimate of the rate of fractionation, $\dot{M}$, is obtained by multiplying the dimensionalized upward flow rate at the wall, $\mu = 0$, by the boundary layer thickness δ_c and by the circumference of the magma chamber. Assuming a cylindrical magma chamber of volume V and height ℓ, one finds

$$\dot{M} \sim 2\sqrt{\pi}\,\zeta(0)\,\lambda(1-G)\,(R_b V/LeG\ell)^{1/2} \qquad (30)$$

If one assumes $V = 10^3$ km^3, $\ell = 1$ km, $\lambda = 10^{-12}$ km^2/s, $\zeta(0) = 1$, and

$G \ll 1$, equation (30) reduces to

$$\dot{M} \sim 3.5 \times 10^{-3} \, (R_b/LeG)^{1/2} \ km^3/yr \qquad (31)$$

If the factor in parentheses in (31) is of the order of unity, then $\dot{M} \sim 3.5 \times 10^{-3}$ km^3/yr, which is consistent with inferred rates of fractionation (Smith, 1979; Spera and Crisp, 1981). The parameters values in Table 1 give $R_b \sim 10$. If $G \sim 10^{-2}$ and $Le \sim 10^3$ the desired result is achieved. If $R_b \ll 1$, as would be likely in the case of a high viscosity, silicic magma chamber, or if Le lies in the higher range of possible values, then the rate of fractionation as a result of counterflow in the porous boundary layer may not be sufficient to produce the inferred stratification. This result is consistent with that of Spera et al. (1984) for a viscous boundary layer. These results are based on the assumption that the physical parameters are constant across the layer. Further analysis of the structure of the flow in the porous boundary layer, including the effects of variable viscosity and permeabilty is currently underway.

CONCLUSIONS

In this paper, two problems involving double-diffusive convection in partially molten silicate systems have been considered. The first problem concerned the role of double-diffusive convection during melt production. The calculations showed that, in a growing layer of partial melt, heated from below, in which the compositional gradient was stabilizing, a finite-amplitude instability would set in when the layer was ~800 m thick. If the compositional gradient were destabilizing, compositionally driven convection could occur in a layer only centimeters thick. A qualitative analysis of compositional gradients that might occur in partially molten diapirs suggests that, in some cases, vigorous compositional convection could occur within the partially molten system. The presence of chemical gradients within the partial melt will, depending upon the bulk composition of the source zone, the degree of partial melting, and other factors, tend to homogenize the melt within the source zone. Models of melt-solid segregation such as those of Ahern and Turcotte (1979), Stolper et al. (1981), and McKenzie (1984) should be modified to include double-diffusive effects.

The second problem concerned the structure of a porous boundary layer in a crystallizing magma chamber. If the residual melt fraction had negative compositional buoyancy, or if the negative thermal buoyancy exceeded the positive compositional buoyancy, the boundary layer flow is everywhere directed downward. If, however, the positive compositional buoyancy exceeded the thermal buoyancy, counterflow occurred and a low density liquid would accumulate at the top of the magma chamber. For

low viscosity liquids with relatively rapidly diffusing species, liquid fractionation would occur at a rate of $\sim 10^{-3} km^3/yr$, which is consistent with inferred rates based on field studies. The results of the calculations on the structure of porous boundary layer flow are generally in agreement with those of Spera et al. (1984) for a viscous fluid.

APPENDIX

Onset of Convection in a Chemically Homogeneous, Growing, Partially Molten Layer

The standard way to investigate the onset of thermal convection is to consider a rest-state in which the partial melt layer is in diffusive equilibrium and pressure is hydrostatic. The solutions to the rest-state problem serve as a zeroth order solution, and it is assumed that a linear perturbation analysis gives rise to equations governing a marginally stable steady convective system. The exchange of stabilities occurs at a critical value of a dimensionless group of parameters called the Rayleigh number (Lapwood, 1948). The value of the critical Rayleigh number for a horizontal, fluid-saturated, homogeneous, porous slab subject to a uniform temperature gradient, with temperature decreasing upwards, is $4\pi^2$ (Lapwood, 1948). Here the effects of the non-linear temperature gradient across the layer and the effect of latent heat absorbed at the moving phase boundary are included in the analysis. Physical parameters such as the viscosity, thermal diffusivity, permeability, etc. are assumed constant. The Boussinesq approximation is employed.

The zeroth order problem is:

$$\partial T^o/\partial t = a\, \partial^2 T^o/\partial z^2 \qquad \text{(A1)}$$

where a is the thermal diffusivity $\kappa/\rho c_p$. The boundary conditions are:

$$T^o = T^* \quad \text{at} \quad z = 0 \tag{A2}$$

$$T^o = T_e \quad \text{at} \quad z = h(t) \tag{A3}$$

$$(\rho_\ell \chi L)\, dh(t)/dt = -\kappa\, \partial T/\partial z\Big|_{z = h(t)} \tag{A4}$$

The solid is at T_e everywhere and for all time $t > 0$. The solution to equations (A1)-(A4) is (Carslaw and Jaeger, 1959, Chapter XI):

$$T^o = T^* - \Delta T\ \text{erf}\ (z/2\sqrt{at})/(\text{erf}Q) \tag{A5}$$

where Q is determined from:

$$\sqrt{\pi}\ Q\ \text{erf}Q\ e^{Q^2} = \Delta T\ c_p/\chi L = St$$

and (A6)

$$h(t) = 2Q(at)^{1/2}$$

By using a linearized perturbation analysis on equations (1) through (3) and (5), assuming C is constant, and standard mathematical manipulations, the steady-state convective stability equations are found to be:

$$\nabla^2 w - (K\alpha g/\nu)\ \nabla_h^2 T' = 0 \tag{A7}$$

$$w\partial T^o/\partial z = \lambda\ \nabla^2 T' \tag{A8}$$

∇_h^2 is the horizontal Laplacian. If one now employs a vertical length scale h(t) and writes

$$w = F(z')\ w^t(x,y)$$
$$T' = \Theta(z')\ T^t(x,y) \tag{A9}$$

where $z' = z/h(t)$ and T^t, w^t are periodic functions with dimensionless wavenumber k. Then substitution of (A9) into (A7) and (A8) gives

$$(D^2 - k^2)\ F = -[(\alpha g K k^2)/\nu]\Theta \tag{A10}$$

$$[(-2\Delta TQ/\sqrt{\pi}\ \text{erf}\ Q)\ e^{-Q^2 z'^2}]\ h(t)F/\lambda = (D^2 - k^2)\ \Theta \tag{A11}$$

where $D = d/dz$ and the term in brackets on the left-hand side of (A11) is dT^o/dz'. It has now been assumed that λ and a are equal.

One then scales the temperature as $\Theta = \Theta'\Delta T2Q/\sqrt{\pi}\text{erf}Q$ and the velocity as $F = F'\lambda/h(t)$ to obtain:

$$(D^2 - k^2)F' = -k^2 R'\Theta' \tag{A12}$$

$$(D^2 - k^2)\Theta' = -e^{-Q^2 z'^2}\ F' \tag{A13}$$

where

$$R' = \frac{\alpha g K \Delta T h(t)}{\nu\lambda}\ \left(\frac{2}{\sqrt{\pi}}\ \frac{Q}{\text{erf}Q}\right)$$

Equations (A12) and (A13) are subject to boundary conditions

$$\Theta' = 0 \quad \text{at} \quad z' = 0,1$$
$$F' = D^2F' = 0 \quad \text{at} \quad z' = 0,1 \tag{A14}$$

It should be noted that equations (A12), (A13), and (A14) no longer contain time explicitly.

In order to solve the eigenvalue problem represented by equations (A12)-(A14), one notes that for a reasonable range of values of the Stefan number $c_p\Delta T/ L$, $Q < 1$ (Carslaw and Jaeger, 1959, p. 287). Therefore $\exp(-Q^2z^2) \sim 1-Q^2z^2$, where, for simplicity, the primes have been dropped. The non-constant nature of the conductive temperature gradient makes a closed form solution difficult. Following Chandresekhar (1961) and deriving an approximate solution, the functions

$$F = \sum_{n=1}^{\infty} C_n \sin n\pi z \tag{A15}$$

which satisfy the boundary conditions are substituted into (A13) and solved for Θ. One finds:

$$\Theta = -\sum_{n=1}^{\infty} \frac{C_n}{(n^2\pi^2+k^2)} \left\{ \left[Q^2z^2 + \frac{2Q^2}{(n^2\pi^2+k^2)} - \frac{8n^2\pi^2Q^2}{(n^2\pi^2+k^2)^2} - 1 \right] \sin n\pi z \right.$$

$$\left. + \frac{4n\pi Q^2 z}{(n^2\pi^2+k^2)} \cos n\pi z - \frac{4\pi nQ^2(-1)^n}{(n^2\pi^2+k^2)\sinh\lambda} \sinh\lambda z \right\} \tag{A16}$$

Upon substituting (A15) and (A16) into (A12), one obtains a set of homogeneous equations for the expansion coefficients C_n. The condition that not all $C_n = 0$ gives a condition on R' that

$$R' \cong \frac{(\pi^2+k^2)^2}{k^2(1-Q^2[(1/3)-(1/2\pi^2)])} \tag{A17}$$

Upon defining the Rayleigh number R as R' ($\sqrt{\pi}$ erfQ/2Q), (A17) leads to

$$R = \frac{\alpha g K \Delta T h(t)}{\nu \lambda} = \frac{(\pi^2+k^2)}{k^2}\left(1 - \frac{Q^2}{2\pi^2}\right) \qquad \text{(A18)}$$

$$R_c = 4\pi^2 \left(1 - Q^2/2\pi^2\right) \qquad \text{(A19)}$$

NOMENCLATURE

a thermal diffusivity, $\kappa/\rho c_p$

b grain diameter

C concentration of diffusing chemical component

$\overline{C}$ reference concentration

C^* similarity constant

C_w concentration at magma-country rock contact in porous boundary layer

C_∞ reference concentration in magma chamber interior

c_p specific heat of solid/melt mixture

c_ℓ specific heat of melt

D mass of diffusivity of chemical component

$f'(\mu)$ dimensionless, vertical velocity function in porous boundary layer

G buoyancy ratio

g acceleration due to gravity

$h(t)$ thickness of growing, partially molten layer

J source of chemical component in melt

K permeability

k dimensionless wavenumber

K_∞ reference permeability in porous boundary layer

L latent heat

Le Lewis number, λ/D

ℓ height of magma chamber, scale height

$\dot{M}$ rate of liquid fractionation due to counterflowing in porous boundary layer

P pressure

Pr Prandtl number

Q dimensionless parameter in Stefan problem

R thermal Rayleigh number for layer heated from below

R^* compositional Rayleigh number for a layer heated from below

R_b thermal Rayleigh number based on boundary layer parameters

R_b^* compositional Rayleigh number based on boundary layer parameters

R_c critical thermal Rayleigh number for layer heated from below

R^f critical thermal Rayleigh number for onset of finite-amplitude instability in double-diffusive layer

Sc Schmidt number

St Stefan number

T temperature

T' perturbation temperature

$\overline{T}$ reference temperature

T^* basal temperature of growing, partially molten layer

T^o zeroth order temperature

T_e eutectic temperature, starting temperature for solid layer melted by heating from below

T_w temperature at magma-country rock contact in porous boundary layer

T_∞ reference temperature in magma chamber interior

t time

u Darcian velocity vector

V volume of magma chamber

v horizontal velocity in porous boundary layer

w vertical velocity in porous boundary layer

x,y,z Cartesian coordinates, z vertically upward

α coefficient of thermal expansion

β coefficient of compositional expansion

$\Delta T = T^* - T_e$

δ_C compositional boundary layer thickness

δ_T thermal boundary layer thickness

ζ parameter describing variation in viscosity and permeability in porous boundary layer

η dynamic viscosity

θ dimensionless temperature

κ thermal conductivity of solid-melt mixture

λ thermal diffusivity, $\kappa/\rho_\ell C_\ell$

μ similarity variable

ν kinematic viscosity

ν_∞ reference kinematic viscosity

ρ density of solid-melt mixture

ρ_ℓ melt density

ϕ dimensionless concentration

χ melt fraction or porosity

ACKNOWLEDGMENTS

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REFERENCES

Ahern, J. L., and Turcotte, D. L., 1979. Magma migration beneath an ocean ridge. Earth Planet. Sci. Lett., 45:115-122.

Arndt, N. T., 1977. Ultrabasic magmas and high-degree melting of the mantle. Contrib. Mineral. Petrol., 64:205-221.

Batiza, R., and Vanko, D., 1984. Petrology of young Pacific seamounts. J. Geophys. Res., 89:11235-11260.

Bowen, N. L., 1915. The crystallization of haplobasaltic, haplodioritic and related magmas. Am. J. Sci., 40:161-185.

Carslaw, H. S., and Jaeger, J. C., 1959. Conduction of Heat in Solids. 2nd ed., Clarendon Press, Oxford, 510 p.

Chen, C. F., and Turner, J. S., 1980. Crystallization in a double-diffusive system. J. Geophys. Res., 85:2573-2593.

Cheng, P., and Minkowycz, W. J., 1977. Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike. J. Geophys. Res., 82:2040-2044.

Coriell, S. R., Cordes, M. R., Boettinger, W. J., and Sekerka, R. F., 1980. Convective and interfacial instabilities during unidirectional solidification of a binary alloy. J. Cryst. Growth, 49:13-28.

Davis, B. T. C., and Schairer, J. F., 1965. Melting relationships in the join diopside-forsterite-pyrope at 40 kilobars and at one atmosphere. Carnegie Inst. Washington Year Book, 64:123-126.

Hildreth, W., 1981. Gradients in silic magma chambers: Implications for lithospheric magmatism. J. Geophys. Res., 86:10153-10192.

Huppert, H. E., and Sparks, R. S. J., 1984. Double-diffusive convection due to crystallization in magmas. Ann. Rev. Earth Planet. Sci., 12:11-37.

Huppert, H. E., Sparks, R. S. J., and Turner, J. S., 1983. Laboratory investigations of viscous effects in replenished magma chambers. Earth Planet. Science Lett., 65:377-381.

Huppert, H. E., and Turner, J. S., 1981. Double-Diffusive convection. J. Fluid Mech., 106:299-330.

Irvine, T. N., 1980. Magmatic infiltration metasomatism, double-diffusive fractional crystallization, and cumulative growth in the Muskox intrusion and other layered intrusions, in Physics of Magmatic Processes, ed. by R. B. Hargraves, Princeton University Press, Princeton, N.J., p. 325-384.

Kerr, R. C., and Turner, J. S., 1982. Layered convection and crystal layers in multicomponent systems. Nature, 298:731-733.

Lapwood, E. R., 1948. Convection of a fluid in a porous medium. Proc. Cambridge Phil. Soc., 44:508-521.

Lowell, R. P., 1982. Thermal convection in magmas generated by hot-plate heating. Nature, 300:253-254.

Marsh, B. D., and Kantha, L. H., 1978. On the heat and mass transfer from an ascending magma. Earth Planet. Sci. Lett., 39:435-443.

McBirney, A. R., 1980. Mixing and unmixing of magmas. J. Volc. and Geotherm. Res., 7:357-371.

McBirney, A. R., Baker, B. H., and Nilson, R. H., 1984. Liquid fractionation, Part I: Basic principles and experimental simulations. J. Volc. Geotherm. Res., (in press).

McBirney, A. R., and Noyes, R. M., 1979. Crystallization and layering of the Skaergaard intrusion. J. Petrol., 20:487-554.

McKenzie, D., 1984. The generation and compaction of partially molten rock. J. Petrology, 25:713-765.

Mo, X., Carmichael, I. S. E., Rivers, M., and Stebbins, J., 1982. The partial molar volume of Fe_2O_3 in multicomponent silicate liquids and the pressure dependence of the oxygen fugacity in magmas. Mineral. Mag., 45:237-245.

Nield, D. A., 1968. Onset of thermohaline convection in a porous medium. Water Resources Res., 4:553-560.

Nilson, R. H., and Baer, M. R., 1982. Double-diffusive counterbuoyant boundary layer in laminar natural convection. Int. J. Heat Mass Transfer, 25:285-287.

Nilson, R. H., McBirney, A. R., and Baker, B. H., 1984. Liquid fractionation, Part II: Fluid dynamics and quantitative implications for magmatic systems. J. Volc. Geotherm. Res., (in press).

Presnall, D. C., Dixon, S. A., Dixon, J. R., O'Donnell, T. H., Brenner, N. L., Schrock, R. L., and Dycus, D. W., 1978. Liquidus phase relations on the join diopside-forsterite-anorthite from 1 atm to 20 kbar: Their bearing on the generation and crystallization of basaltic magma. Contr. Mineral. Petrol., 66:203-220.

Ribe, N. M., 1984. The generation and composition of partial melts in the Earth's mantle. EOS, 65:1141.

Rudaiah, N., Srimani, P. K., and Freidrich, R., 1982. Finite amplitude convection in a two-component fluid saturated porous layer. Int. J. Heat Mass Trans., 25:715-722.

Sleep, N. H., 1974. Segregation of magma from a mostly crystalline mush. Geol. Soc. America Bull., 85:1225-1232.

Smith, R. L., 1979. Ash-flow magmatism, Geol. Soc. Am. Special Paper 180, 5-28.

Sparrow, E. M., Lee, L., and Shamsunder, N., 1976. Convective instability of a melt layer heated from below. J. Heat Transfer, 98:88-94.

Spera, F. J., and Crisp, J. A., 1981. Eruption volume, periodicity, and caldera area: Relationships and inferences on developments of compositional zonation in silicic magma chambers. J. Volc. Geotherm. Res., 11:169-187.

Spera, F. J., Yuen, D. A., and Kemp, D. V., 1984. Mass transfer rates along vertical walls in magma chambers and marginal upwelling. Nature, 310:764-767.

Spera, F. J., Yuen, D. A., and Kirschvink, S. J., 1982. Thermal boundary layer convection in silicic magma chambers: Effects of temperature dependent rheology and implications for thermogravitational chemical fractionation. J. Geophys. Res., 87:8755-8767.

Stolper, E., Walker, D., Hager, B. H., and Hays, J. F., 1981. Melt segregation from partially molten source regions: The importance of melt density and source region size. J. Geophys. Res., 86:6261-6271.

Turcotte, D. L., and Ahern, J. L., 1978. A porous flow model for magma migration in the asthenosphere. J. Geophys. Res., 83:767-772.

Turner, J. S., 1973. Buoyancy Effects in Fluids. Cambridge University Press, New York, 367 p.

Turner, J. S., and Gustafson, L. B., 1981. Fluid motions and compositional gradients produced by crystallization on melting at vertical boundaries. J. Volcanol. Geotherm. Res., 11:93-125.

Veronis, G., 1965. On finite amplitude instability in thermohaline convection. J. Marine Res., 23:1-17.

Waff, H. S., 1980. Effects of the gravitational field on liquid distribution in partial melts within the upper mantle. J. Geophys. Res., 85:1815-1825.

Walker, D., Stolper, E. M., and Hays, J. F., 1978. A numerical treatment of melt/solid segregation: Size of the eucrite parent body and stability of the low velocity zone. J. Geophys. Res., 83:6005-6013.

Weill, D. F., Hon, R., and Navrotsky, A., 1980. The igneous system $CaMgSi_2O_6$-$CaAl_2Si_2O_8$-$NaAlSi_3O_8$: Variations on a classic theme by Bowen, in Physics of Magmatic Processes, ed. by R. Hargraves, Princeton University Press, Princeton, N.J., pp. 48-92.

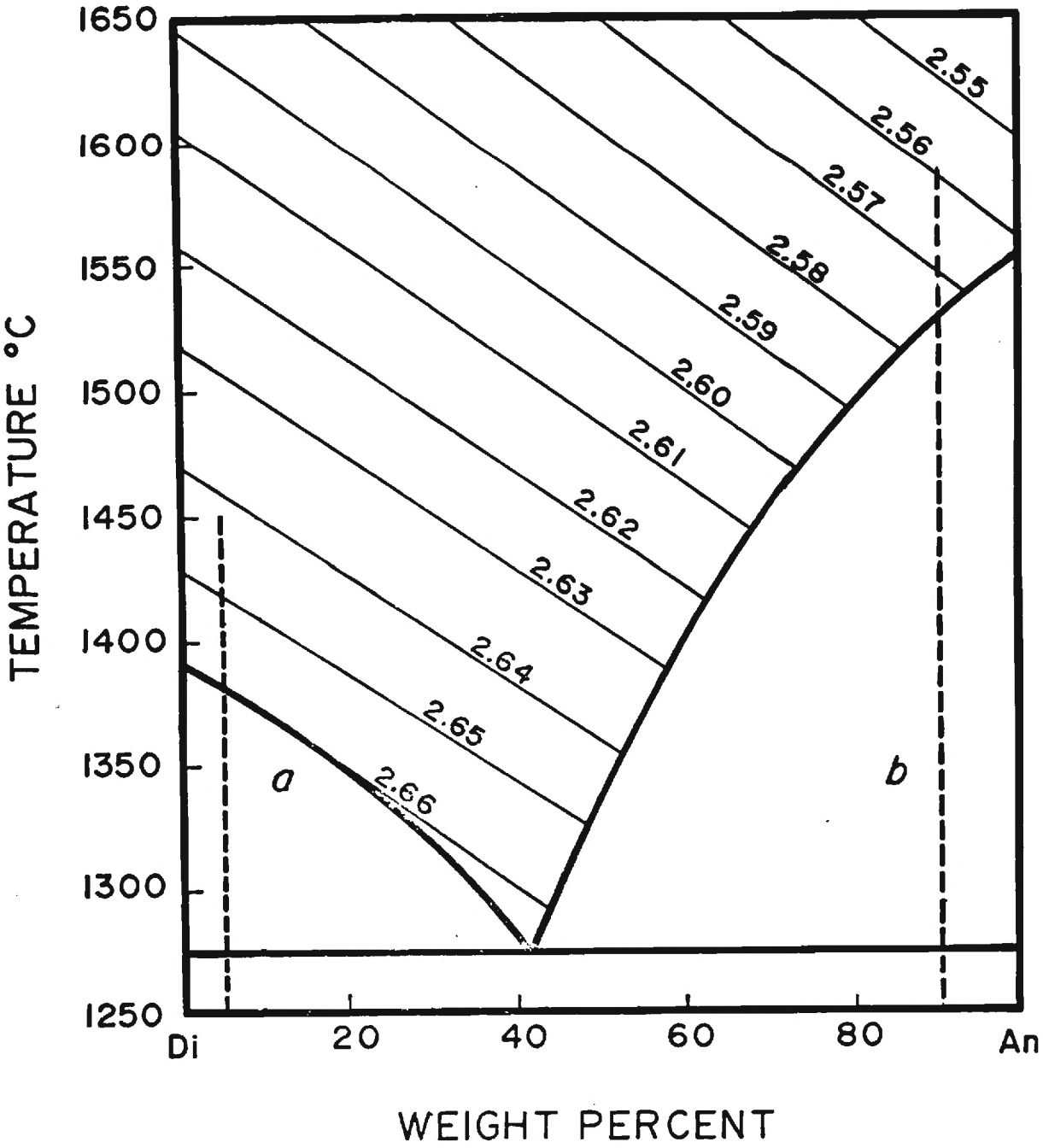
Yoder, H. S., Jr., 1976. Generation of Basaltic Magma. Nat. Acad. of Sciences, Washington, DC, 265 p.

Yoder, H. S., Jr., 1978. Basic magma generation and aggregation. Bull. Volcanol., 41:301-316.

TABLE 1. Values of Parameters

Parameter	Value	Comments
g	10^3 cm/s^2	
λ	10^{-2} cm^2/s	typical igneous rock value
D	10^{-5}-10^{-9} cm^2/s	range of diffusivities in silicate liquids
χ	0.1	assumed values
b	$10^{-1/2}$ cm	assumed values
ν	10 cm^2/s	typical "basaltic" melt viscosity

LIST OF FIGURES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1250
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2.57
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2.59
2.60
2.61
2.62
2.63
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a
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Di
20
40
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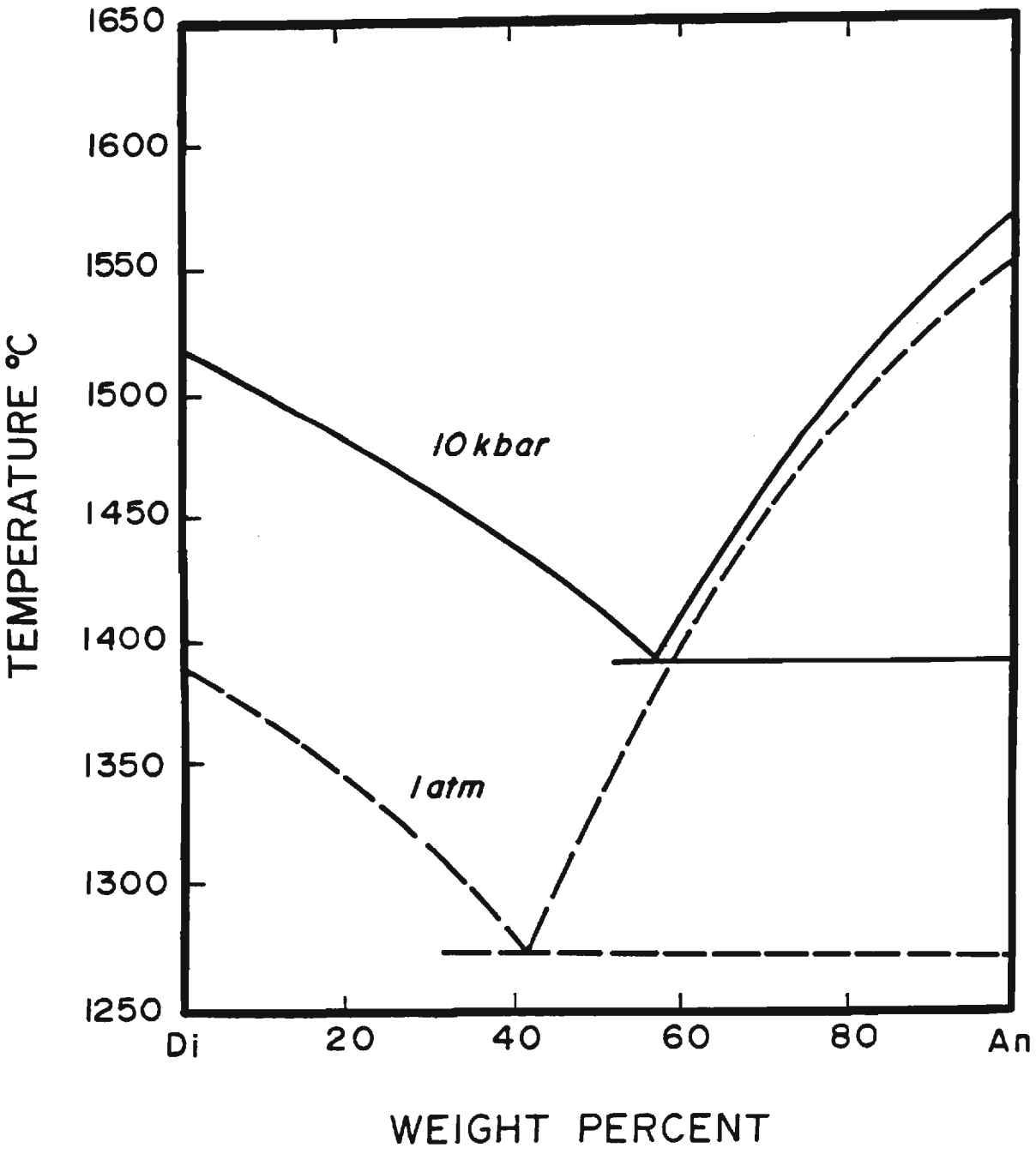
80
An
WEIGHT PERCENT

FIGURE 1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1600
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TEMPERATURE °C
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40
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80
An
WEIGHT PERCENT

FIGURE 2

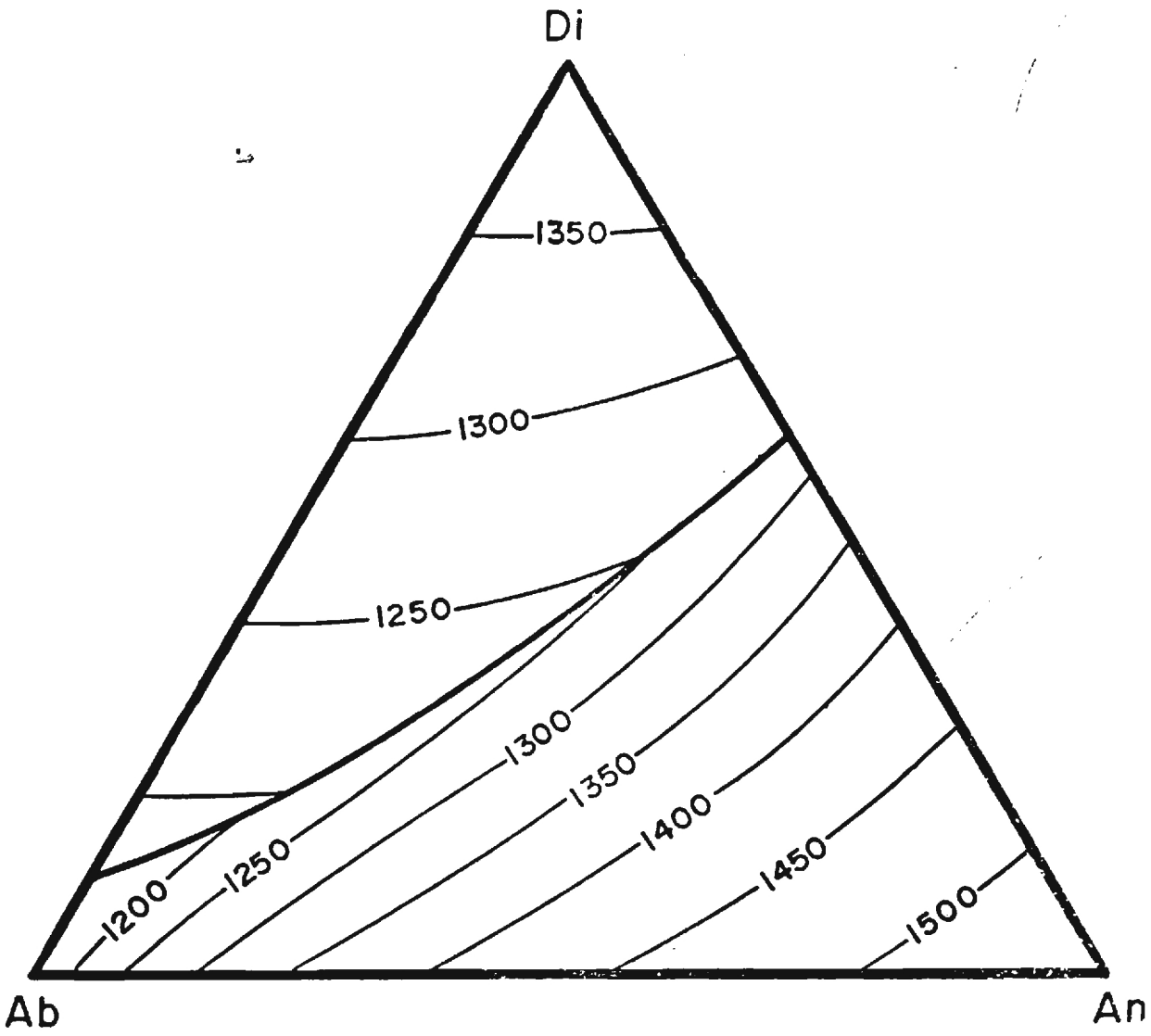


FIGURE 3a

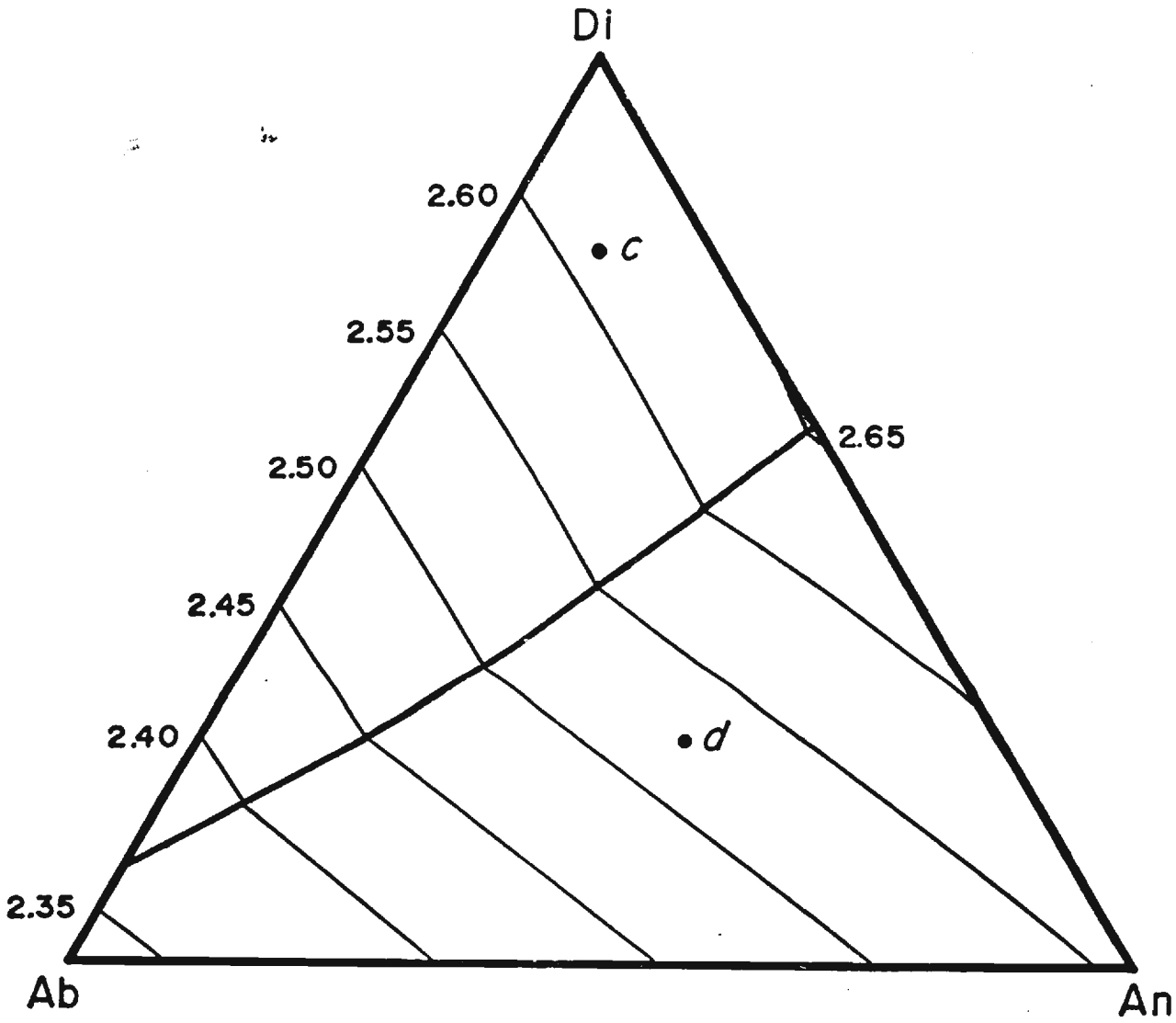
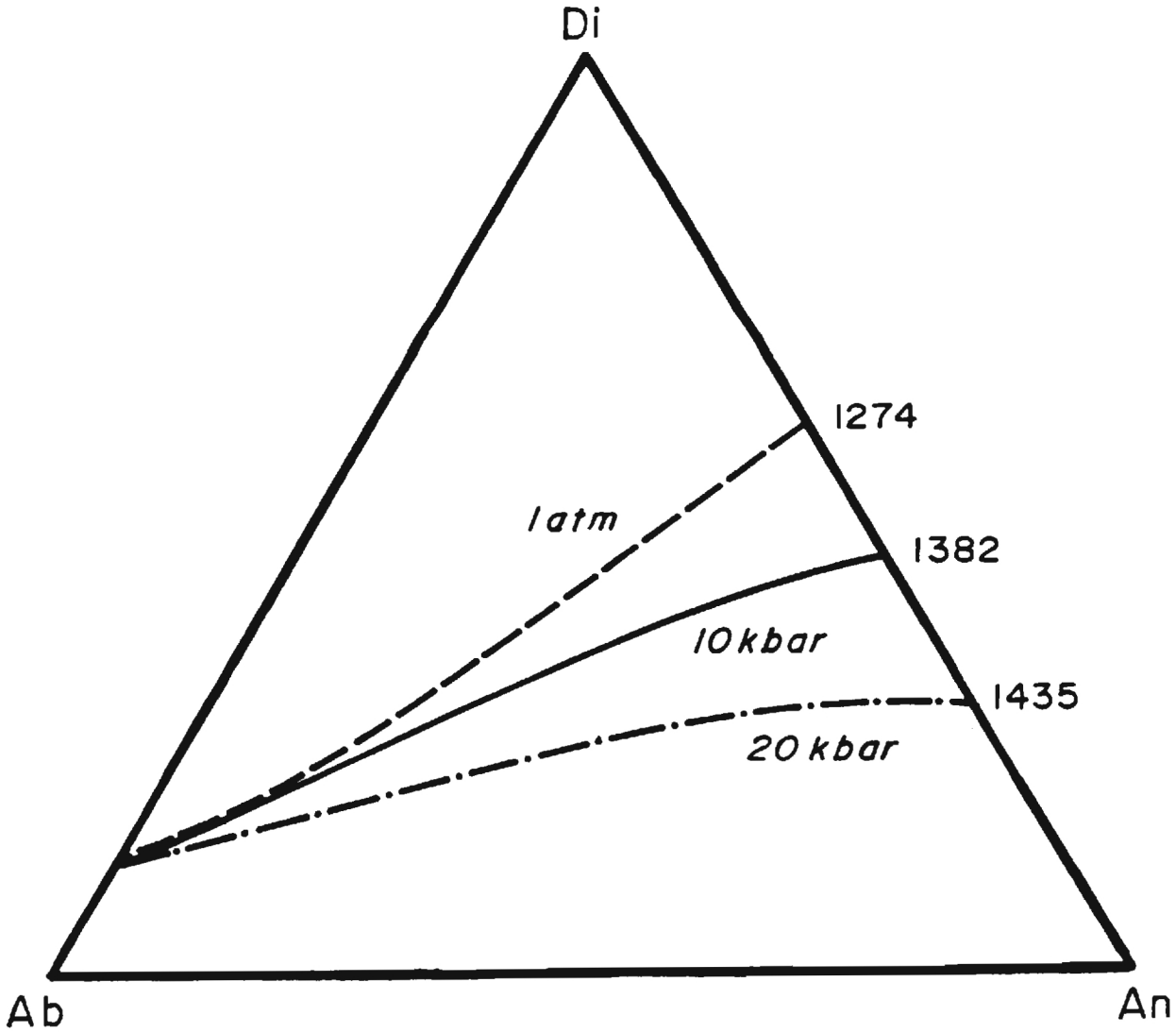


FIGURE 3b



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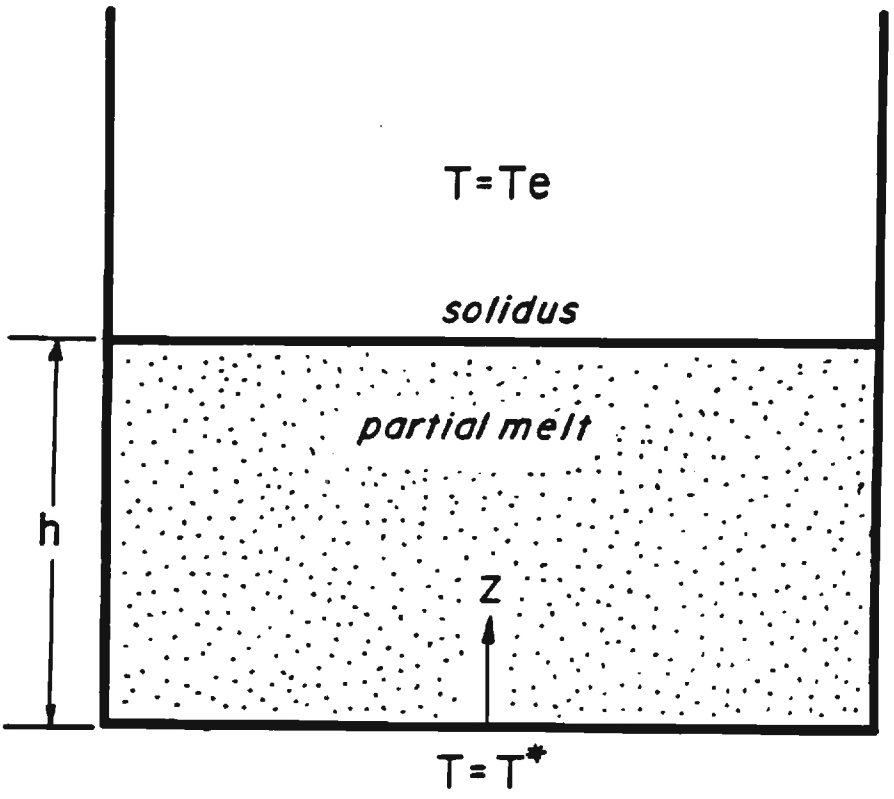

FIGURE 5

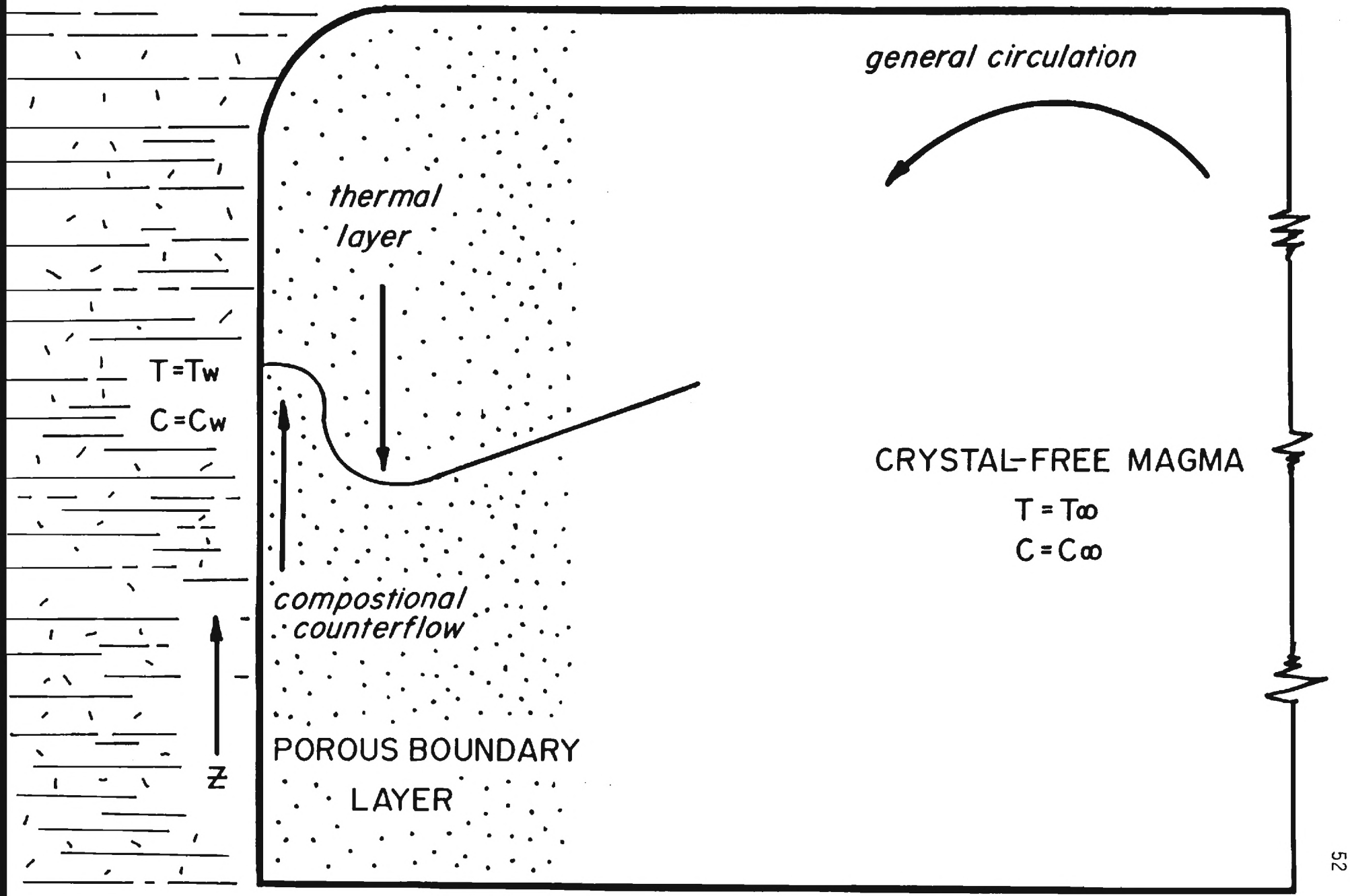
general circulation
thermal
layer
T=Tw
C=Cw
CRYSTAL-FREE MAGMA
T = T∞
C = C∞
compostional
counterflow
POROUS BOUNDARY
LAYER
z

FIGURE 6

HYDROTHERMAL MODELS
FOR THE GENERATION OF MASSIVE SULFIDE ORE DEPOSITS

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ABSTRACT

The discovery of massive sulfide ore deposits at certain sites at active seafloor spreading centers provides a basis for considering the energetics of processes that have concentrated similar large ore deposits that have been preserved in volcanic rocks on land. In this paper we construct transport models based on hydrothermal convection systems in the oceanic crust. We investigate models in which the circulation is driven by heat extracted from the permeable crustal rocks themselves as well as by heat conducted through the roof of a replenished, vigorously convecting crustal magma chamber. If the hydrothermal fluids carry 100 ppm dissolved iron as found in black smokers on the East Pacific Rise and exit at 350^{o}C or greater, calculations show that the heat content of the oceanic crust is insufficient to account for ore deposits of 3MT or greater. Heat extraction through the roof of a magma chamber, in conjunction with the formation of a thin (~1 km) layer of plated gabbro, may account for ore bodies of the order of 3MT, provided the permeability of the oceanic crust in the magma-hydrothermal fluid contact zone is 10^{-15} m^2 or less. If the conducting boundary layer were only one meter thick, locally, for 10^3 years, the permeability could be as high as 10^{-13} m^2. The permeability in the discharge zone could, conceivably, be two orders of magnitude greater than the permeability at depth. More work is needed on the details of the effects of magma chamber replenishment and off-axis dike injection and on the role of magmatic fluids. Very large ore deposits (i.e., ~100MT) are probably formed in multi-stage events.

INTRODUCTION

The discovery of massive copper-iron-zinc sulfide deposits at sites at seafloor spreading centers in the Red Sea (Miller et al., 1966; Hunt et al., 1967; Degens and Ross, 1969; Backer and Schoell, 1972), on the East Pacific Rise (CYAMEX Scientific team, 1979; RISE Project Group, 1980; Hekinian et al., 1983), the Galapagos Spreading Center (Malahoff et al., 1983), and the Juan de Fuca Ridge (Koski et al., 1984; Koski et al, in press; Tivey and Delaney, in press), provides a basis to consider the energetics of processes that have concentrated large, ancient massive sulfide deposits preserved in volcanic rocks on land. In this paper we construct transport models based on subseafloor hydrothermal convection systems employing reasonable assumptions of heat supply, solution chemistry, and time duration derived from investigations of actual high-temperature hydrothermal systems at seafloor spreading centers. Our calculations show that ore deposits in the 3 million ton range can be formed as a result of hydrothermal circulation that extracts heat across the thin conductive boundary layer at the top of a vigorously convecting, replenished magma chamber. The results of the calculations also indicate that much larger massive sulfide deposits are the composite product of multiple episodes of hydrothermal activity, and that some of our assumptions, based on field observations, may require significant changes to explain the largest deposits.

SIZE OF VOLCANIC-HOSTED MASSIVE SULFIDE DEPOSITS

At this early stage when less than one percent of the global length of seafloor spreading centers has been systematically explored, it is unlikely that the largest massive sulfide deposits in ocean basins are yet known (Rona, 1983, 1984). The largest hydrothermal deposit so far discovered at a seafloor spreading center is the Atlantis II Deep deposit of the Red Sea containing 32.5MT (1MT = 10^6 metric tons = 10^9 kg) of metals as sulfides and other mineral phases (estimated bulk brine-free tonnage 83MT) in metalliferous sediments (Table 1; Bischoff and Manheim, 1969; Backer, 1979; Schott, 1980; Saudi-Sudanese Red Sea Joint Commission, 1982; Nawab, 1984). The Atlantis II Deep deposits represent the optimum efficiency in concentration of a hydrothermal deposit at a spreading center by precipitation and entrapment of particulate hydrothermal mineral phases in a hypersaline density-stratified brine contained within a basin (Bischoff, 1969). Another hydrothermal deposit at a seafloor spreading center that may be large is a deposit apparently consisting of coalesced solid massive sulfide mounds and chimneys at the Galapagos Spreading Center with bulk tonnage of 10MT tentatively estimated from surface sampling only (Malahoff et al., 1983). Precipitation of massive sulfides from high-temperature solutions venting through chimneys as "black smokers" at oceanic ridges under conditions of unrestricted oceanic circulation is considered to be relatively inefficient as a process for concentrating large massive sulfide deposits because a significant fraction of the metals are dispersed as suspended particulate matter by mid-depth oceanic circulation (Rona, 1984;

Hekinian et al., 1983a,b). Small deposits consisting of massive sulfide mounds each with bulk tonnage up to several thousand metric tons have been found at sites on the East Pacific Rise (CYAMEX, 1979; RISE Project Group, 1980; Hekinian et al., 1983a,b).

In comparison with recent massive sulfide deposits at seafloor spreading centers, the tonnage of ore in large ancient massive sulfide deposits hosted in the basaltic portion of ophiolitic crustal sections inferred to have been generated by seafloor spreading in ocean basin or arc-related geologic settings ranges up to about 30MT (Figure 1; Mosier et al., 1983). This tonnage is of the same order of magnitude as the largest deposits known from present seafloor spreading centers (Table 2). Ore tonnage in ancient massive sulfide deposits hosted in andesitic and rhyolitic volcanic rocks representing continental or island arc geologic settings ranges up to about 250MT, an order of magnitude greater than the largest such deposits known in basaltic rocks. The tonnages cited (Figure 1; Tables 1 and 2) represent the total massive sulfide deposit, which is generally constructed of multiple individual lenses or pods each related to discrete depositional events or episodes.

BASIC REQUIREMENTS FOR MASSIVE SULFIDE ORE FORMATION

The problem of the formation of massive sulfide ore deposits has been the subject of several recent studies (e.g., Solomon, 1976; Spooner, 1977; Parmentier and Spooner, 1978; Strens and Cann, 1982; Cathles, 1983). Elder (1977) provides of a discussion of the genesis of hydrothermal ore deposits in general, and some of his results are applicable to the present study. Rather than review the ore genesis problem in detail, we will focus on the fundamental requirements for and constraints on the generation of "volcanogenic" massive sulfide ores, particularly as may pertain to ores formed in conjunction with hydrothermal processes now known to exist at oceanic spreading centers (see, for example, Rona and Lowell, 1980; Cathles, 1981; Fyfe and Lonsdale, 1981; MacDonald et al., 1980).

The formation of a massive ore deposit on the ocean floor indicates that there are anomalous, localized circumstances that occur within the general setting of the hydrothermal system. Such circumstances may be the existence of favorable channels for the venting of high temperature ore fluids (Solomon, 1976), as well as an ambient oceanic enviornment suitable for the deposition of the ore from the venting fluids (Solomon and Walshe, 1978; Cathles, 1983; Rona, 1984). Though these factors are of considerable importance, we will assume the existence of suitable channels and a favorable depositional enviornment. All of the metal in solution will be assumed to go into the formation of a massive FeS_2 ore deposit (i.e., none of the ore solution will be dispersed into the ambient ocean and none will be disseminated as stockwork deposits beneath the sea floor). We will also not examine the process by which

the metals are extracted from the source rocks, but rather will simply assume that the fluid carries 100 ppm Fe, as found in black smokers on the East Pacific Rise (Edmond, 1981). (The iron concentration in the lower brine of the Atlantis II deep in the Red Sea, where the temperature is 56°C and the salinity is 319°/∞, ranges from 90 to 107 ppm (Brooks et al., 1969). The metal concentration in the high temperature fluids at depth are, therefore, probably greater in the Red Sea hydrothermal system than in the East Pacific Rise system.) We will further assume that for deposition of massive sulfide ore that the venting fluid must be at 350°C or greater (see Parmentier and Spooner, 1978; Strens and Cann, 1982).

In this paper we will center our analysis on the energetics of a hydrothermal system: the magnitude and nature of the heat source, the rate of extraction of heat by the circulating fluid, and the time of operation of the system. In this sense, the energetics of ore formation is synonymous with the energetics of the hydrothermal system itself. Studies of the energetics of high temperature hydrothermal systems on land will therefore provide useful insight into the energetics of ore forming processes.

It appears to be universally accepted that the heat source for both continental and submarine high temperature hydrothermal systems is essentially of magmatic origin. In some cases, the magmatic character of the heat source is not directly important. Deep seated magma bodies may merely act to heat the overlying crustal rocks. There may be an impermeable zone between the magma chamber and the permeable, heated crustal layer. The hydrothermal circulation may then extract heat from

the crustal rocks without tapping the heat of the underlying magma chamber. Most calculations on hydrothermal ore forming processes either implicitly or explicity assume the circulation system is able to extract the heat of the magma (Solomon, 1976; Parmentier and Spooner, 1978; Elder, 1977; Cathles, 1983). Strens and Cann (1982) have shown that the heat content of the upper 1 kilometer of the oceanic crust is insufficient to drive a hydrothermal system that can account for ore deposits such as those listed in Tables 1 and 2. They invoke a magmatic heat source to explain the deposits (see also Cann and Strens, 1982).

Another factor to be considered is the volume from which heat is extracted. Individual massive sulfide ore bodies rarely exceed 500m in horizontal extent. The question arises as to whether the venting fluid has extracted heat only from rocks nearby and subjacent to the ore body, or whether the venting fluid has collected heat from a volume that has lateral dimensions several times the lateral dimensions of the ore body. Elder (1977) and Solomon (1976), for example, consider heat extraction from magma bodies that have surface areas of 10^2 km^2 or more and are several kilometers thick. Strens and Cann (1982), on the other hand, favor a fault-controlled circulation system that extracts heat from an area of rather small horizontal extent.

A third factor to be considered is the nature of permeability in the oceanic crust and the best hydrothermal circulation model. Even though field data from both continental and marine hydrothermal systems suggest that the fluid flow is significantly modified, if not controlled, by discrete fault and fracture zones (e.g., Grindley, 1965; Hose and Taylor, 1974; Bailey et al., 1976; Swanberg, 1975; RISE, 1980; Green

et al., 1981; Bodvarsson, 1982; Rona et al., 1984), the approximation of Darcian flow through a porous medium has often been used to model the hydrothermal flow (e.g., Ribando et al., 1976; Fehn and Cathles, 1979; Goyal and Kassey, 1980; Fehn et al., 1983). With regard to the formation of massive sulfide ore deposits, Solomon (1976), Parmentier and Spooner (1978), and Cathles (1983) use porous medium models, whereas Strens and Cann (1982) use a discrete fracture model.

The final concern is the time of operation of the hydrothermal system. Spooner (1977) suggests that an individual massive sulfide deposit in Cyprus formed in a time of 10^5 years. This time is roughly in agreement with the estimate of Solomon (1976), who also gives a time of 10^6 years to form the very large Rio Tinto deposit (230MT of ore, Mosier et al., 1983). Cathles (1983), however, calculates that the individual sulfide lenses of the Kuroko deposits formed in less than 5000 years and perhaps in as little as 100 years. Observations on the rate of heat transfer in sulfide-laden black smokers venting at the East Pacific Rise indicate vent lifetimes of 1 to 10 years (Macdonald et al., 1980). The lifetimes of isolated geothermal systems in New Zealand (Elder, 1977) and Iceland (Bodvarsson, 1982) are thought to be of the order of 10^4 years. These estimates are based in part on the rate at which hydrothermal circulation cools an intrusive magma body (Elder, 1977).

Although it is generally accepted that hydrothermal deposits form during the evolution of a transient hydrothermal episode (e.g., Parmentier and Spooner, 1978), there has been relatively little work done on explicitly time-dependent hydrothermal systems. Lowell (1975,

1976) and Strens and Cann (1982) have examined time-dependent heat transfer in circulation through a large-scale fracture system. These investigators have considered heat extraction from the hot country rocks adjacent to a fracture, but neither model has considered the process of heat transfer from a subjacent magma chamber. Cathles (1981, 1983) has developed models of hydrothermal convection of an unreplenished cooling magmatic pluton. In his models, the pluton was considered to be solid (T=700°C) and to cool by conduction and hydrothermal circulation through the regions that were permeable. His models have not considered the cooling of a fluid, convecting, replenished magma chamber. The heat transfer by conduction across an impermeable boundary layer between the magma chamber and the overlying hydrothermal system has not been investigated thoroughly.

In this paper, we will develop time-dependent hydrothermal circulation models that will yield further insight into the physics of heat extraction from the permeable rocks of the oceanic crust and from a subjacent magma chamber. The models will be highly idealized, but they will have the advantage of being amenable to a mathematical formulation that permits analytical solutions. The relationships between the heat source volume, the thickness and areal extent of the conducting boundary layer, crustal permeability, temperature and rate of fluid flow, and time are elucidated. To limit the number of possible variables, the calculations will be based on the following assumptions:

[1] A model ore deposit will be assumed to contain 3×10^9 kg (i.e., 3MT) FeS_2, corresponding to 10^9 kg of Fe.

[2] The concentration of Fe in solution is 10^{-4}, as measured in certain black smokers.

[3] The temperature, T_w, of the exiting solution must be greater than 350°C.

[4] All of the metal in solution goes into forming the massive sulfide deposit.

From the above assumptions, several system constraints can be immediately derived. Assumptions [1] and [2], for example, imply that for the model ore deposit, the mass m of hot water to be vented at the sea floor is $m = 10^{13}$ kg. This result, together with assumption [3], gives the total heat, H_F, that must be transferred by the exiting fluid. Given that the specific heat of water s is 8×10^3 j/kg °C (Straus and Schubert, 1977), then

$$H_F = m\ s\ T_w = 2.8 \times 10^{19}\ j \qquad (1)$$

A complete list of symbols used in this paper is given in the Appendix.

HYDROTHERMAL MODELS

We will consider two hydrothermal models. In the first model, we will consider only a crustal heat supply. We carry out these calculations in spite of the result of Strens and Cann (1982) mainly because their flow model consists of a single longitudinal fracture that is extremely inefficient at extracting heat. By considering sets of fractures (i.e., the fluid descends in one fracture set and ascends in another) a more efficient heat extracting system is set up because there is greater contact area between the circulating fluids and the country rock. Secondly, some important features of the hydrothermal systems are elucidated. Our calculations do strongly confirm the conclusion of Strens and Cann as to the inadequacy of a crustal heat source. In the second model we consider a hydrothermal system which extracts heat from the underlying magma chamber by conduction across a thin boundary layer. Our calculations will show that such a system may produce the model ore deposit in a physically reasonable fashion provided the conducting boundary layer is less than ~1 km thick and the permeability of the crustal rocks in the contact zone near the top of the magma chamber is 10^{-15} m^2 or less. If the conductive boundary layer is kept exceptionally thin (~1 m), perhaps as a result of magma chamber replenishment, greater permeabilities are permissible.

A. Crustal Heat Supply

To investigate the rate of heat extraction from the oceanic crustal rocks, we consider a simple model based on forced convective circulation through a set of discrete, vertical fractures that are placed parallel

to the strike of the ocean ridge axis. The model has been used effectively to study heat extraction from a hot, dry, rock continental geothermal reservoir (Gringarten et al., 1975; Lowell, 1976). The model is somewhat unrealistic, geologically and physically, but because fracture length and spacing are treated as adjustable parameters, one is able to design an optimum situation for massive sulfide deposition as a result of heat extraction from the crustal rocks. Since we are able to show that the crustal heat source is highly unlikely to be sufficient, even in an optimum situation, our results confirm the conclusion given by Strens and Cann (1982).

In the model, cold water is assumed to circulate downward to a depth h and to enter a discrete fracture or a system of parallel, discrete, vertical fractures through which the water ascends. At time $t = 0$, the water enters the fracture or fractures at temperature T_{wo} and ascends at constant mass flow rate q per unit fracture length. The crustal rocks in which the fracture or fractures are embedded are initially at a uniform high temperature T_{Ro}. As the water ascends up the fracture or fractures, heat is conducted from the rock to the water and the temperature of the water increases. The first water exits the fracture system at the sea floor at a temperature $T_w(h,t) = T_{Ro}$, but as heat is mined from the rock, the temperature of the exiting fluid declines. When the exit temperature drops below 350^oC, massive sulfide deposition will be assumed to cease.

Given a Cartesian coordinate system at the base of the fracture with x directed perpendicular to the plane of the fracture and z directed positively upward, the basic equations for heat transfer by conduc-

tion from impermeable rock to water flowing in a flat, thin fracture are (Bodvarsson, 1969; Lowell, 1976):

$$\partial^2 T_R/\partial x^2 = (1/a)\partial T_R/\partial t \tag{2}$$

$$sq\partial T_w/\partial z = 2\lambda(\partial T_R/\partial x)\Big|_{x=0} \tag{3}$$

$$T_R(x,z,0) = T_{Ro} \tag{4}$$

$$T_w(x,0,t) = T_{wo} \tag{5}$$

$$T_R(z,t) = T_w(z,t) \tag{6}$$

where T_R, T_w are the rock and water temperatures, respectively, a is the thermal diffusivity of the rock and λ its thermal conductivity, and s is the specific heat of water. Equation (2) represents the diffusion of heat in the impermeable rock and it has been assumed that heat conduction perpendicular to the fracture is much greater than heat conduction in the direction of fluid flow. Equation (3) states that the rate of heat transfer by advection in the fracture equals the rate at which heat is conducted through both sides of the rock-fracture interface. Since the fracture is thin, the temperature is assumed to be constant across the width of the fracture. Equations (4) and (5) give the initial rock temperature and the water temperature at the base of the fracture, respectively. Equation (6) states that there is thermal equilibrium between the water flowing in the fracture and the rock temperature at the rock-fracture interfaces. The solution to equations (2)-(6) can be found in a rather straightforward manner (Bodvarsson, 1969). The temperature of the exiting fluid $T_w(h,t)$ is

$$T_w(h,t) = T_{wo} + (T_{Ro} - T_{wo}) \text{ erf } [\lambda h/\text{sq}(at)^{1/2}] \qquad (7)$$

where erf represents the error function. Since for small argument (i.e., large values of t) erf $x \sim x$, equation (7) shows that the exit temperature decays at $t^{-1/2}$ for large t. Given values of the pertinent parameters in (7), one is able to determine the flow rate q which will yield an exit temperature greater than some specified value for a time t. Equation (7) shows that the lower the flow rate, the longer the system can operate above a specified exit temperature. Moreover, equation (7) can also be shown to yield the important result that, the lower the rate flow, the greater the total yield of hot water above a specified temperature.

We rewrite (7) by multiplying and dividing the arguments of the error function by an assumed fracture length ℓ. Then $q\ell=Q$, the mass discharge in kg/s and $h\ell=A$, the water-rock contact area for the discharging system. By taking reasonable values for the physical parameters, we calculate the water-rock contact area needed to yield the 10^{13} kg of water required to generate a 3MT sulfide ore deposit as a function of time. Table 3 gives the results. By assuming an average depth of 2 km for the hydrothermal system in the crust, the required fracture length is A/2. Thus a 3MT ore deposit forming in 10^3 years would require more than 30 km of discharging fracture length, whereas the same deposit forming in 10^5 years would require only 3 km. It is clear, however, that individual sulfide lenses or pods are formed by local discharges. Individual deposits have a typical planform area of 1 km^2 or so (Franklin et al., 1981), so that if 30 km of fracture length

is required, then we might envision a discharge system of 30 fractures each 1 km long and spaced roughly 30 meters apart. The exit temperature calculation was based on thermally isolated fractures, yet if the spacing D = 30 m and t = 10^3 years, the Fourier number, $N_F = at/D^2$, is 30. Thus there would be severe thermal interference between fractures and the exit temperature could not be maintained.

If one allowed the deposit to form by a slow discharge from a single 3 km long fracture over a long time interval of 10^5 years, the assumption of thermal isolation would require that the recharge system be more than 10 km away from the discharging fracture. Such a fortuitous arrangement seems highly unlikely. Block faulting occurs on a finer scale (Macdonald, 1982, 1983a, b), and there is little data to support the idea that sulfide ore deposits are generated by a low level discharge over a long time (i.e., black smokers discharge at $\sim 10^2$ kg/sec for considerably shorter times (Converse et al., 1984)).

A final argument that can be made against the crustal heat source stems from the assumption of a forced convective circulation. By controlling the circulation rate one is able to maximize the heat extracted from a given volume of rock. In the natural situation, the circulation is controlled by thermal buoyancy forces. In the free-convective model, both the flow rate and the exit temperature decrease with time (Lowell, 1975). Since the total mass yield of hot water as a function of time decreases with increasing flow rate, the high mass flow rate that occurs early in the life of a free-convective system, when the buoyancy forces are large, severely limits the mass of hot water that can exit the hydrothermal system above the prescribed temperature. The limits on

high-temperature mass transport by a free-convection system driven by time-dependent thermal conduction through impermeable rocks will be treated in more detail in section B on magmatic heat supply.

The principal result of this section has been to show that even given the most ideal circumstances it is not possible to account for the generation of massive sulfide ores of significant magnitude by hydrothermal heat extraction from rocks of the oceanic crust.

B. Magmatic Heat Supply

In this section we examine a hydrothermal system driven by heat transferred from a subjacent magma by conduction through a thin, impermeable boundary layer. This model is considerably different from other models involving the cooling of a pluton by hydrothermal circulation (Cathles, 1981, 1983) because in Cathles' models the pluton is considered to be an unreplenished magma chamber that cools by conduction only. In our model, the magma chamber is considered to be vigorously convecting and subject to replenishment from below. Our model is consistent with the idea that as differential magma chambers solidify, crystals accumulate on the floor. The magma chamber mainly crystallizes from the floor upwards rather than from the roof downwards (Pallister and Hopson, 1981).

The nature of the boundary layer between a magma chamber and an overlying hydrothermal system defies a simple physical/mathematical description. The boundary layer is undoubtedly highly non-uniform in space and time. Bjornsson et al. (1982) suggest that water penetration to within a few meters, or possibly even less than one meter, of a magma body is responsible for the high thermal output of the Grimsvötn geothermal area in Iceland.

Pallister and Hopson (1981), however, interpret the high-level gabbros of the Samail ophiolite to have formed by plating on the roof of the magma chamber. These plated gabbros range in thickness up to 800 meters, and Lister (1983) has argued that they act as an impermeable conductive boundary layer between the magma chamber and the hydrothermal systems operating in the overlying crustal rocks. According to

Pallister and Hopson (1981), the Samail ophiolite magma chamber was repeatedly replenished at the ridge axis and had a lifetime of the order of 10^5 years; the plated gabbros thicken away from the ridge axis and in some places the gabbros are cut by dikes, which must have been emplaced after the formation of the plated layer. The occurrence of "upper border zone" rocks that were apparently formed by freezing downward from the roof of layered intrusions such as the Skaergaard (Wager and Brown, 1968) and the Stillwater complex (Todd et al., 1982) also suggests that the boundary layer may grow to be hundreds of meters thick.

Because the boundary layer is too complex for us to model its effect on hydrothermal circulation in detail, we will construct a fairly simple model that elucidates the basic physics of the problem and permits an order of magnitude estimate as to the efficacy of the system for the generation of massive sulfide ore deposits. In this analysis, we will neglect heat extraction from the crust and assume that at $t = 0$ a fracture system suddenly opens to the top of the convecting magma chamber. Cold water descends, is heated by thermal conduction across the thin boundary layer separating the magma chamber and the fractured crust, ascends and exits at the sea floor, carrying metals leached from the fractured crustal rocks. For simplicity, we neglect heat losses in the ascending fluid. We treat the dynamics of the hydrothermal system as Darcy flow in a porous medium, but circulation in a discrete large-scale fracture system can be accommodated with relatively minor modification.

The vertical mass flow rate per unit area of the porous medium, q_z, results from a balance between viscous and buoyant forces. Following

Bodvarsson and Lowell (1972) and Cathles (1981) we write:

$$q_z \cong \rho_f \alpha g K T_w/\nu \tag{8}$$

where ρ_f is the density of water, α is the thermal expansion coefficient of water, g the acceleration due to gravity, K the permeability of rock, ν the kinematic viscosity of water, and T_w, as before, is the temperature of the ascending fluid. It is assumed that the descending fluid is at 0^oC. To express conservation of energy, we simply assume that all the heat conducted through the roof of the magma chamber is advected away by the ascending hydrothermal fluid. Thus,

$$s\, q_z\, T_w = H \tag{9}$$

where H is the rate of heat conduction through the boundary layer separating the magma chamber and the hydrothermal system. In order to consider hydrothermal circulation through widely spaced, narrow, discrete, vertical fractures, one substitutes $d^3/24$ for K in (8) and multiplies the right-hand side of (9) by ℓ^*, where d is the fracture width and ℓ^* is the horizontal distance between the recharging and discharging fractures (see Bodvarsson and Lowell, 1972). In equations thus modified, q_z becomes the mass flow per unit length of the fracture. We now consider two models for the rate of heat loss by conduction through the top of the magma chamber.

1. Conductive Cooling of a Uniform Temperature Solid

If it is assumed that the magma body is initially at a uniform temperature T_s, and heat is lost by conduction through a plane interface

maintained at temperature T_w, the heat flux H is (Carslaw and Jaeger, 1959, p. 60):

$$H = \lambda(T_s - T_w) \; / \; \sqrt{\pi a t} \tag{10}$$

Since T_w is a function of time, (10) is not a rigorously correct expression. Equation (10) does give the correct time dependence, and the numerical error that is introduced by the approximation is certainly less than the uncertainty in other physical parameters of the system (e.g., the permeability may vary over several orders of magnitude).

Upon substituting (8) and (10) into (9), we obtain:

$$s\rho_f \; \alpha \; g \; K \; T_w^{\,2} \; / \nu \; = \lambda(T_s - T_w) \; / \; \sqrt{\pi a t} \tag{11}$$

The solution to the quadratic equation for T_w (11) is

$$T_w = \{-\lambda/\sqrt{\pi a t} + (\lambda/\sqrt{\pi a t})[1 + 4T_s\sqrt{\pi a t}\;\rho_f s \alpha g K/\lambda\nu]^{1/2}\}/\;[2s\rho_f \alpha g K/\nu] \tag{12}$$

A binomial expression for small values of the time shows

$$T_w \sim T_s - \theta(t) \tag{13}$$

where $\theta(t)$ is a small term proportional to t. For large values of time, however, (12) yields:

$$T_w \sim (\lambda T_s \nu / s K \alpha \rho_f g)^{1/2} \; (\pi a t)^{-1/4} \tag{14}$$

For the equation (14) to hold, we require that:

$$[4T_s \rho_f s \alpha g K/\lambda\nu] \; (\pi a t)^{1/2} \gg 1 \tag{15}$$

Letting $T_s = 10^{3o}C$, $\rho_f = 500$ kg/m^3, $s = 8 \times 10^3$ j/kg-^oC, $\alpha = 1.6 \times 10^{-4}/^oC$, $g = 10$ m/sec^2, $\lambda = 2.5$ w/m-^oC, $\nu = 10^{-7}$ m^2/sec, $a = 10^{-6}$ m^2/sec, we obtain:

$$t > 3 \times 10^{-23}/K^2 \text{ sec} \qquad (16)$$

where K has units of m^2.

The permeability of the upper few kilometers of the oceanic crust is quite uncertain. Estimates range from high values in the range $K = 10^{-8}$ to 10^{-10} m^2 based on Lister's (1974) thermal contraction theory and estimates of crack width and spacing in basalts (Johnson, 1980) to ~2×10^{-15} m^2 measured at a depth of ~200 m in pillow basalts near the Costa Rica Rift (Anderson and Zoback, 1982). Even if the permeability is as low as 10^{-15} m^2, equation (14) would govern the rate of decay of the exit temperature after ~1 year of circulation.

For most of the life of the convection system, the exit temperature falls off as the one-fourth power of time, and (8) shows that the flow rate has the same time dependence. These results are consistent with the numerical calculations of Lowell (1975), which show that the flow rate and temperature in a fracture-loop hydrothermal system, driven by thermal convection across a fracture interface, decay with time in the same manner.

We then use (14) to estimate the time, τ, at which the exit temperature falls below 350^oC, and we can then integrate (8) over the time interval from 0 to τ to determine the total mass flow of hydrothermal fluid at temperature greater than 350^oC. Upon substituting the same values as were used previously, we obtain:

$$\tau \cong 3.2 \times 10^{-20}/K^2 \quad \text{sec} \tag{17}$$

$$Q = \int_0^{\tau} q_z dt \cong 1.2 \times 10^{-4}/K \quad \text{kg/km}^2 \tag{18}$$

For K in the range 10^{-10} - 10^{-15} m^2, $\tau \cong 3.2$ - 3.2×10^{10} sec, and $Q = 1.2 \times 10^6$ - 1.2×10^{11} kg/km^2. To obtain the output of 10^{13} kg required for a 3MT ore deposit, the thermal plume must extract heat through the upper surface of a magma chamber that is 10^7 km^2 if the permeability is 10^{-10} m^2. If the permeability is only 10^{-15} m^2, 10^2 km^2 of surface area is required. A typical massive sulfide lens occupies ~1 km^2, and the actual discharge zone may be even smaller. Therefore, these calculations, because they show heat extraction from a very large surface area is required, indicate that massive sulfide ore deposits cannot be generated by hydrothermal circulation that is driven by conductive heat transport across a thickening boundary layer at the top of a cooling magma body.

2. Formation of the Plated Gabbro Layer and Heat Transfer Due to Convection in the Magma Chamber

In the preceding calculation, we neglected the effect of latent heat of freezing in the formation of the boundary layer; and we ignored the fact that the magma body is transporting heat upward by convection. Lister (1983) has argued that the boundary condition at the top of an ocean ridge magma chamber such as the Samail ophiolite is one of constant heat flux. As heat is transported upward by the convecting magma it is carried away by the overlying hydrothermal system. Lister calculated that the plated gabbro layer of ~0.5 km is approximately the

correct thickness so that the equilibrium is maintained. Presumably, the plated gabbro layer formed by a transient process, whereby the hydrothermal system initially extracted more heat than was supplied by the convecting magma. The extra heat extraction came from the latent heat of freezing the plated layer. An appropriate model for the heat and mass transport by the hydrothermal system during the formation of the plated gabbros involves an analysis of the heat transport across a boundary layer that thickens in time due to the freezing of material. The rate of growth of the plated gabbro can be approximated by the classical Stefan-Neumann solution (Carslaw and Jaeger, 1959, p. 285; Irvine, 1970).

We assume that the magma chamber is at its melting temperature T_L and the boundary at $z=0$ is at $T=T_w$. Since T_w is a function of time, the Stefan problem cannot be solved in closed form. With the same impropriety as in equation (10), however, we find that the heat flux H_1 into the hydrothermal system, due to the formation of plated gabbros, is approximately:

$$H_1 \cong \lambda(T_L - T_w) \,/\, \mathrm{erf}\gamma \sqrt{\pi a t} \tag{19}$$

where γ is a parameter determined from

$$\gamma \,\mathrm{erf}\gamma \, e^{\gamma 2} = c(T_L - T_w) \,/\, \sqrt{\pi}\, L \tag{20}$$

The specific heat of the magma is c, and L is the latent heat of fusion of the plated gabbro. The location of the freezing front Z(t) is

$$Z(t) = 2\gamma \sqrt{at} \tag{21}$$

If we assume $c = 10^3$ j/kg-^oC, $L = 4 \times 10^5$ j/kg, $T_L = 1200^oC$, then so long as $(T_L - T_w) > 100^oC$, $0.3 < \gamma < 0.9$. In this range of γ, $\text{erf}\gamma \sim \gamma$ so the heat flux given by (19) is at most a factor of 3 times the value given by (10), in which latent heat was neglected.

We now also take into account the heat transferred by thermal convection within the magma chamber, H_o. For a vigorously convecting magma chamber, the additional heat transfer due to convection is expressed by the Nusselt number, $N \sim (R/R_C)^{1/3}$, where R is the Rayleigh number and R_C is the critical value for the onset of thermal convection. If we neglect complicating factors such as double-diffusive effects (e.g., Huppert and Sparks, 1984), $R_C \sim 10^3$. The Rayleigh number for a convecting magma chamber has been estimated to be quite large, e.g., 10^{14} (Elder, 1977), 10^{15} (Lister, 1983). Lister's value gives $N \sim 10^4$, and if we assume a superadiabatic temperature gradient of 0.5^oC/km, within the magma chamber, then with $\lambda = 2.5$ watts/m-^oC,

$$H_o \cong (2.5)(5 \times 10^{-4})(10^4) = 12.5 \text{ watts/m}^2 \qquad (22)$$

Tht total heat flux, H, that is transported by the hydrothermal system is

$$H = H_o + H_1 \qquad (23)$$

Upon substituting (23) into (9) and employing (8), we again obtain an expression for the exit temperature of the hydrothermal fluid. The "large time" approximation still holds in the same manner as discussed previously. We obtain, therefore,

$$T_w \sim (\nu/\rho_f s\alpha gK)^{1/2}[(\lambda T_L/\text{erf}\gamma\ \sqrt{\pi at}) + H_o]^{1/2} \qquad (24)$$

From (24), we again obtain the time τ during which $T_w \geq 350^oC$. By assuming that $\text{erf}\gamma = 0.5$, and $T_L = 1200^oC$ and that the other parameters have the same values as used previously, we obtain

$$\tau = 1.15 \times 10^3 / [1.57 \times 10^{16} K - H_o]^2 \text{ sec} \qquad (25)$$

where K has units of m^2 and H_o has units of watts/m^2. Equation (25) has the property that for fortuitous values of K and H_o, $\tau \to \infty$. This simply means that for these particular values, the exit temperature will asymptotically approach $T_w = 350^oC$. A negative value of τ simply implies that T_w will always be above 350^oC for the given values of K and H_o. The value of the exit temperature will then be given by (24) as $\tau \to \infty$. For the value $H_o = 12.5$ watts/m^2 given by (22), the value of the permeability that gives $\tau = \infty$ is $K \cong 8 \times 10^{-16}$ m^2. This value is somewhat less than permeability usually given for the oceanic crust, but it is not a totally unrealistic value. On the other hand, the value of H_o given by (22) is possibly a little high for a steady-state value due to convection within the magma chamber. Lister's (1983) own estimate is about 7.5 watts/m^2, and Sleep and Wolery (1978) estimate that only 1 watt/m^2 is advected through crust less than 10^6 years old. It is difficult to imagine that H_o is more than 25 watts/m^2, and yet K may be several orders of magnitude greater than values of the order of 10^{-15} m^2 that yield values to approaching infinity. For most of the range of permeability values that are estimated for the oceanic crust, equation (25) provides a reasonable approximation to the time of operation of the high temperature hydrothermal system.

We assume H_o = 12.5 watts/m^2 and calculate the total mass flux per unit area Q that exits the hydrothermal system at the required temperature by substituting (24) into (8) and integrating q_z over the interval from 0 to τ. The results, for different values of the permeability, are shown in Figure 2. Figure 2 shows the rapid rate of decline in the production of high temperature hydrothermal fluid as the permeability increases. If the permeability is 10^{-13} m^2, for example, Q = 2.4 x 10^9 kg/km^2. Nearly 5 x 10^3 km^2 of contact area would be required in order to produce 10^{13} kg of water at $T_w \geq 350^oC$. If K = 10^{-15} m^2, however, Q = 6.6 x 10^{12} kg/km^2 and less than 2 km^2 of contact area are required to give the desired result. For K = 10^{-15} m^2, the time of operation of the hydrothermal system, from equation (25), is approximately 4 x 10^4 years. In this time interval, the plated gabbro layer would become ~1 km thick. These numbers are all quite reasonable. Therefore, the model indicates that, if the permeability of the oceanic crust is 10^{-15} m^2 or less, a hydrothermal system, driven by a combination of heat transfer from a vigorously convecting magma chamber and heat transfer from freezing a thickening layer of gabbro plated to the roof of the magma, is able to transport enough high temperature water to generate a massive sulfide ore deposit of 3MT. The model further shows, however, that, if the permeability is ~10^{-14} m^2 or greater, the temperature and mass flow rate of the hydrothermal system decay too rapidly to generate an ore body of appreciable size.

We point out that these values for the permeability refer to the permeability in the region near the top of the magma chamber where heat extraction is taking place. After the fluid has been heated by contact

with a sufficient surface area at the top of the magma chamber and ascends, the flow may be focused into a somewhat narrower discharge zone by entering a region of higher permeability. One can envision a focusing factor of 100, for example, so that a fluid extracting heat from 2 km^2 at the top of a magma chamber may discharge through zone 200 m x 100 m. In order to maintain conservation of fluid mass, the permeability in the discharge zone would be two orders of magnitude greater than in the zone of heat extraction from the magma chamber.

The question still remains as to whether the formation of an ore deposit is evidence for rather low permeability of the oceanic crust in the neighborhood of the magma chamber, or whether a more efficient mechanism of heat transfer is possible so that high temperature discharge is maintained even if the permeability at depth is much greater than 10^{-15} m^2. One must also question how very large deposits (~100MT) are generated.

DISCUSSION

To address in detail the question of whether massive sulfide ore deposits can occur in regions where the oceanic crust is highly permeable at depth would lead beyond the scope of this paper; however, some qualitative discussion is in order. It is interesting to note that other models for the generation of massive sulfide ore deposits, though based on somewhat different physical assumptions than ours, suggest rather low permeabilities (Parmentier and Spooner, 1978; Cathles, 1983). In these models, low permeability was "built into" the calculations, whereas in our model, the permeability was a free parameter that could be varied over a considerable range. Only if the permeability in the neighborhood of the subsurface magma chamber were in the low end of the acceptable range did our calculations yield a reasonable sized ore body. Because our model is so oversimplified, however, it is useful to examine some factors which may argue towards the possibility of higher permeability systems.

We first of all point out that replenishment of the magma chamber is insufficient, of itself, to give rise to greater heat transport to the hydrothermal system. In fact, replenishment of the magma chamber is implicit in the value H_o = 12.5 watts/m^2 in equation (22). If the magma chamber had a volume of 500 km^3 and an upper surface area of 100 km^2 (roughly in accord with the Samail ophiolite (Pallister and Hopson, 1981)), the latent heat content would be approximately 6 x 10^{20} joules, assuming a latent heat of 4 x 10^5 j/kg and a density of 3 x 10^3 kg/m^3. If that energy is lost only by conduction through the roof at a rate of

12.5 watts/m^2, then the magma chamber would be solidified in 5 x 10^{11} sec. Since we assumed that the rate of heat transport was maintained for nearly that same time as a result of vigorous convection (Nusselt number = 10^4), the magma chamber must have been kept mostly liquid during that time interval. Thus to have the heat flux assumed in equation (22) implicitly assumes that heat is continually supplied from below by replenishment of the magma chamber. We point out further, though, that we have assumed an average heat flux in equation (22). The heat flux at the axis may have been higher than in the off axis regions. Such a lateral variation int he heat flux would be consistent with the observation that the plated gabbro layer thickens towards the margins of the Samail ophiolote (Pallister and Hopson, 1981). The conduction boundary layer may therefore be thinner near the axis. This may be important because it is the thickening of the boundary layer with time, thus reducing the rate of heat flux to the hydrothermal system, that causes the decline in q_z and T_w. If, for example, we apply equations (8) and (9) to a case in which the boundary layer is held at constant thickness δ and the magma at constant temperature T_L, then

$$T_w \cong (T_L \nu / \rho_f s \alpha g K \delta)^{1/2} \tag{26}$$

$$q_z \cong (\lambda T_L \rho_f \alpha g K / s \nu \delta)^{1/2} \tag{27}$$

Equation (26) yields

$$\delta = 7.6 \times 10^{-4} / K \quad m \tag{28}$$

using the same values as before. If $K = 10^{-13}$ m^2, $\delta = 0.76$ m, which is still slightly more than that estimated by Bjornsson et al. (1982) for

water penetration near the boundary of the Grimsvötn magma chamber. If the mass flux from such a system was maintained for 10^3 years, the total mass flux would be 1.7×10^{13} kg/km^2, so that less than 1 km^2 of contact area would be needed to generate a 3MT ore deposit. The effect of high heat flux into the axial zone of an ocean ridge magma chamber must be examined more rigorously. Another observation that may be important is that dikes have apparently cut the plated gabbro layer. Such dike injection may serve to rejuvenate a dying hydrothermal system by providing additional heat and by furnishing pathways for penetration of hydrothermal fluids to the top of the magma chamber. Finally, we reemphasize that these calculations have been based on the assumption that the concentration of metals is 100 ppm. If the metal concentration were greater, as appears to be the case for the Red Sea brines, for example, the crustal permeability could be increased in the model calculations.

The hydrothermal systems that we have considered have dealt with heat transfer from a liquid, convecting magma chamber. After the magma chamber has solidified, there is still a significant amount of heat stored within it. As our calculations have shown, equations (10) through (18), this heat cannot be tapped by conduction through a thickening conductive boundary layer. If the solidified magma chamber is penetrated and cooled by hydrothermal circulation (e.g., by downward propagation of fractures (Lister, 1974; Bodvarsson, 1982) or as suggested by Cathles (1983)), then a greater source of heat is available. Hydrothermal cooling of the solidified magma chamber suggests the idea that ore deposits form in two, or perhaps multiple, stage events. The

early stage occurs during initial circulation within the crustal rocks and the extraction of heat from the still active, replenished magma chamber. The early stage may have multiple events if replenishment does not exactly keep pace with hydrothermal heat withdrawal. The late stage may occur when hydrothermal fluids penetrate the solidified magma chamber. Upon considering that our calculations are based on a 3MT model deposit, one might readily consider that large deposits are multi-stage products.

That a single deposit may be formed by multiple events does not seem unreasonable, even in a seafloor spreading environment. At a spreading rate of ~1 cm/yr, crustal material generated at the ridge axis is still within the median valley after 10^6 years. If high temperature hydrothermal activity has a periodicity of ~10^4 years (Rona et al., 1984), the material within the median valley may be subject to as many as 100 cycles of hydrothermal activity. Even at a fast-spreading ridge, material within 10 km of the ridge axis may undergo as many as 10 cycles, if the periodicity is the same as suggested for slow-spreading ridges.

CONCLUSIONS

We have developed models of hydrothermal massive sulfide ore deposits that may be formed in basaltic volcanic rocks at an oceanic spreading center environment. Our models treated both heat extraction from permeable rocks of the oceanic crust and heat extraction across the conductive boundary layer at the roof of a replenished, vigorously convecting, mostly liquid magma chamber. We find that the crustal heat source is not sufficient to drive a hydrothermal system that generates ore deposits of 3MT or greater. Heat extraction from the convecting magma chamber, in conjunction with the formation of a thin (~1 km) layer of plated gabbro at the roof of the chamber, may account for ore bodies of the order of 3MT provided the permeability of the oceanic crust where heat is extracted from the top of the magma chamber by the hydrothermal circulation is 10^{-15} m^2 or less.

Large massive sulfide deposits with bulk tonnages in the range of 10 to 80MT hosted in basaltic oceanic crust are probably composites of sulfide lenses or pods formed by multiple events and multi-stage episodes of hydrothermal activity. Two general stages of high temperature hydrothermal activity may occur--one during the active, vigorously convecting stage of a magma chamber, the second one during hydrothermal cooling of the solidified pluton. More work is needed on the details of both of these stages. Very large massive sulfide deposits with bulk tonnages up to 250MT hosted in andesitic and rhyolitic volcanic rocks involve additional processes. The effects of magma replenishment, dike

injection, and the contribution of metals transported by magmatic fluids need to be analyzed more fully. If it is possible for the conductive boundary layer to be as thin as one meter, locally, for a time of 10^3 years, then the permeability need be 10^{-13} m^2 or less. The permeability in the discharge zone could conceivably be two orders of magnitude greater than the permeability at depth.

APPENDIX

LIST OF SYMBOLS

a = thermal diffusivity of the oceanic crust

A = $h\ell$ = contact area at water-rock interface

c = specific heat of magma

d = fracture width

D = fracture spacing in the discharge zone

g = acceleration due to gravity

h = depth of fracture system in the oceanic crust

H = rate of heat conduction across the impermeable boundary layer separating the magma chamber from the hydrothermal system

H_F = total heat transfer by exiting hydrothermal fluid

H_o = heat transferred to hydrothermal system as result of convection in magma chamber

H_1 = heat transferred to hydrothermal system as a result of freezing at top of magma chamber

K = permeability of the oceanic crust

ℓ = fracture length

ℓ^* = distance between discharge and recharge zones in fracture-loop system

L = latent heat of fusion of plated gabbro

m = total mass of hot water needed for a 3MT ore deposit

N = Nusselt number

N_F = Fourier number, at/D^2

q = mass flow rate of hydrothermal fluid per unit fracture length

q_z = vertical mass flow rate of hydrothermal fluid per unit area of heat extraction at top of magma chamber

Q = total mass discharge of hydrothermal system per unit area of heat extraction from magma chamber

R = Rayleigh number

R_c = critical Rayleigh number

s = specific heat of hdyrothermal fluid

t = time

T_L = temperature of liquid, convecting magma chamber

T_R = temperature of the oceanic crust

T_{Ro} = initial temperature of oceanic crust

T_s = temperature of magma chamber that is cooling by conduction

T_w = temperature of hydrothermal fluid

T_{wo} = initial temperature of hydrothermal fluid

Z = location of freezing front

α = thermal expansion coefficient of water (hydrothermal fluid)

γ = parameter governing rate of advance of freezing front near top of magma chamber

δ = assumed boundary layer thickness in vigorously convecting magma chamber

λ = thermal conductivity of the oceanic crust

ν = kinematic viscosity of water (hydrothermal fluid)

ρ_f = density of water (hydrothermal fluid)

τ = time at which exiting hydrothermal fluid falls below 350°C

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REFERENCES

Alabaster, T., J. A. Pearce, D. I. J. Mallick, and I. M. Elboushi, The volcanic stratigraphy and location of massive sulphide deposits in the Oman ophiolite, in Ophiolites, edited by A. Panayiotou, pp. 751-757, International Ophiolite Symposium, Nicosia, Cyprus, April 1-8, 1979, Proceedings. Cyprus, Ministry of Agriculture and Natural Resources and Geological Survey Department, p. 751-757, 1980.

Anderson, R. N., and M. D. Zoback, Permeability, underpressures, and convection in the oceanic crust near the Costa Rica Rift Eastern equatorial Pacific, J. Geophys. Res., 87, 2860-2868, 1982.

Bäcker, H., Metalliferous sediments of the Atlantis II Deep, in Offshore Mineral Resources, Orleans, France, B.R.G.M., Document No. 7, 319-338, 1979.

Bäcker, H., and M. Schoell, New deeps with brines and metalliferous sediments in the Red Sea, N. Phys. Sci., 240, 153-158, 1972.

Badham, J. P. N., and P. J. Williams, Genetic and exploration models for sulphide ores in metaophiolites, northwest Spain, Economic Geology, 76, no. 8, 2118-2127, 1981.

Bailey, R. A., G. B. Dalrymple, and M. A. Lanphere, Volcanism structure and geochemistry of Long Valley Caldera, Mono County, California, J. Geophys. Res., 81, 724-744, 1976.

Bischoff, J. L., and F. T. Manheim, Economic potential of the Red Sea heavy metal deposits, in Hot Brines and Recent Heavy Metal Deposits of the Red Sea, edited by E. T. Degens and D. A. Ross, pp. 535-549, Springer-Verlag, New York, 1969.

Bjornsson, H., S. Bjornsson, and Th. Sigurgeirsson, Penetration of water into hot rock boundaries of magma at Grimsvotn, Nature, 295, 580-581, 1982.

Bodvarsson, G., On the temperature of water flowing through fractures, J. Geophys. Res., 74, 1987-1992, 1969.

Bodvarsson, G., Terrestrial energy currents and transfer in Iceland, in Continental and Oceanic Rifts, edited by G. Palmasson, pp. 271-282, Geodynamics Series, v. 8, AGU, Washington, DC, 1982.

Bodvarsson, G., and R. P. Lowell, Ocean-floor heat flow and circulation of interstitial waters, J. Geophys. Res., 77, 4472-4475, 1972.

Brooks, R. R., I. R. Kaplan, and M. N. A. Peterson, Trace element composition of Red Sea geothermal brine and interstitial waters, in Hot Brines and Recent Heavy Metal Deposits of the Red Sea, edited by E. T. Degens and D. A. Ross, pp. 180-203, Springer-Verlag, New York, 1969.

Bugge, J. A. W., Norway, in Mineral Deposits of Europe, Vol. 1, Northwest Europe, edited by A. J. G. Notholt, and M. J. Jones, pp. 199-250, London, Institute of Mining and Metallurgy, 1978.

Cann, J. R., and M. R. Strens, Black smokers fuelled by freezing magma, Nature, 298, 147-149, 1982.

Carslaw, H. S., and J. C. Jaeger, Conduction of Heat in Solids, 2nd ed., Oxford at the Clarendon Press, London, 510 pp., 1959.

Cathles, L. M., Fluid flow and genesis of hydrothermal ore deposits, Economic Geology, 75th Anniversary volume, 424-457, 1981.

Cathles, L. M., An analysis of the hydrothermal system responsible for massive sulphide deposition in the Hokuroku basin of Japan, in The Kuroko and Related Volcanogenic Massive Sulfide Deposits, edited by H. Ohmoto and B. J. Skinner, Economic Geology Monograph 5, 1983.

Converse, D. R., H. D. Holland, and J. M. Edmond, Flow rates in the axial hot springs of the East Pacific Rise ($21^{o}N$): Implications for the heat budget and the formation of massive sulfide deposits, Earth Planet. Sci. Ltrs., 69, 159-175, 1984.

CYAMEX Scientific Team (J. Francheteau, H. D. Needham, P. Choukroune, T. Juteau, M. Seguret, R. D. Ballard, P. J. Fox, W. R. Normark, A. Carranza, D. Cordoba, J. Guerrero, C. Rangin, H. Bougault, P. Cambon, and R. Hekinian), Massive deep-sea sulphide ore deposits discovered on the East Pacific Rise, Nature, 272, 523-528, 1979.

Degens, E. T., and D. A. Ross (editors), Hot Brines and Recent Heavy Metal Deposits in the Red Sea, Springer, New York, NY, 600 p., 1969.

Donoghue, H. G., W. S. Adams, and C. E. Harpur, Tilt Cove copper operation of Maritime Mining Corporation, Ltd., Canadian Mining Metallurgical Bulletin, 52, no. 563, 150-169, 1959.

Douglas, G. V., D. Williams, and O. N. Rove, Copper deposits of Newfoundland, Newfoundland Geological Survey Bulletin, 20, 176 pp., 1940.

Edmond, J. M., Hydrothermal activity at mid-ocean ridge axes, Nature, 290, 87-88, 1981.

Elder, J. W., Models of hydrothermal ore genesis, in Volcanic Processes in Ore Genesis, pp. 4-13, Institution of Mining and Metallurgy and Geological Society, London, 1977.

Fehn, U., and L. M. Cathles, Hydrothermal convection at slow-spreading mid-ocean ridges, Tectonophysics, 55, 239-260, 1979.

Fehn, U., K. C. Green, R. P. Von Herzen, and L. M. Cathles, Numerical models for the hydrothermal field at the Galapagos spreading center, J. Geophys. Res., 88, 1033-1048, 1983.

Franklin, J. M., J. W. Lydon, and D. F. Sangster, Volcanic-associated massive sulfide deposits, Econ. Geol., 75th Anniv. Vol., 485-627, 1981.

Frietsch, R., H. Papunen, and F. M. Vokes, The ore deposits in Finland, Norway, and Sweden: A review, Economic Geology, 74, no. 5, 975-1001, 1979.

Fyfe, W. S., and P. Lonsdale, Ocean floor hydrothermal activity, in The Sea, Vol. 7, edited by C. Emiliani, pp. 589-638, John Wiley, New York, 1981.

Goyal, K. P., and D. R. Karsoy, Fault zone-controlled charging of a liquid-dominated geothermal reservoir, J. Geophys. Res., 85, 1867-1875, 1980.

Green, K. C., R. P. Von Herzen, and D. L. Williams, The Galapagos spreading center at 86°W: A detailed geothermal field study, J. Geophys. Res., 86, 979-986, 1981.

Grenne, T., G. Grammeltvedt, and F. M. Vokes, Cyprus-type sulphide deposits in the western Trondheim district, central Norwegian Caledonides, in Ophiolites, edited by A. Panayiotou, pp. 727-743, International Ophiolite Symposium, Nicosia, Cyprus, April 1-8, 1979, Proceedings, Cyprus, Ministry of Agriculture and Natural Resources and Geological Survey Department, 1980.

Griffitts, W. R., J. P. Albers, and O. Oner, Massive sulphide copper deposits of the Ergani-Maden area, southeastern Turkey, Economic Geology, 67, no. 6, 701-716, 1972.

Grindley, G. W., The geology, structure and exploitation of the Wairakei geothermal field, Taupo, New Zealand, Bull. Geol. Surv. N. Z., 75, 76 pp., 1965.

Gringarten, A. C., P. A. Witherspoon, and Y. Ohnishi, Theory of heat extraction from fractured hot dry rocks, J. Geophys. Res., 80, 1120-1124, 1975.

Hekinian, R., V. Renard, and J. L. Cheminee, Hydrothermal deposits on the East Pacific Rise near 13^{o}N: Geologic setting and distribution of active sulfide chimneys, in Hydrothermal Processes at Seafloor Spreading Centers, edited by P. A. Rona, K. Bostrom, L. Laubier, and K. L. Smith, Jr., pp. 571-594, NATO Conference Series, IV, Marine Sciences, 12, Plenum Press, New York, 1983a.

Hekinian, R., J. Francheteau, V. Renard, R. D. Ballard, P. Choukroune, J. L. Cheminee, F. Albarede, J. F. Minster, J. L. Charlou, J. C. Marty, and J. Boulegue, Intense hydrothermal activity at the axis of the East Pacific Rise near 13^{o}N: Submersible witnesses the growth of a sulphide chimney, Marine Geophysical Research, 6, 1-14, 1983b.

Hose, R. K., and B. E. Taylor, Geothermal systems of northern Nevada, U.S.G.S. Open-File Report 74-271, 1974.

Hunt, J. M., E. E. Hayes, E. T. Degens, and D. A. Ross, Red Sea: detailed survey of hot brine areas, Science, 156, 514-516, 1967.

Huppert, H. E., and R. S. J. Sparks, Double-diffusive convection due to crystallization in magmas, Ann. Rev. Earth Planet. Sci., 12, 11-37, 1984.

Hutchinson, R. W., Genesis of Canadian massive sulphides reconsidered by comparison to Cyprus deposits, Canadian Mining and Metallurgical Bulletin, 58, no. 641, 972-986, 1965.

Irvine, T. N., Heat transfer during solidification of layered intrusions, I. Sheets and sills, Canadian J. Earth Sci., 7, 1031-1081, 1970.

Johnson, D. M., Fluid permeability of oceanic basalts, Init. Rep. Deep Sea Drilling Proj., LI, LII, LIII, part 2, 1473-1477, 1980.

Koski, R. A., D. A. Clague, and E. Ondin, Mineralogy and chemistry of massive sulfide deposits from the Juan de Fuca Ridge, Geol. Soc. Amer. Bull., 95, 930-945, 1984.

Koski, R. A., W. R. Normark, and J. L. Morton, Massive sulfide deposits on the soutehrn Juan de Fuca Ridge: Results of investigations in the USGS study area, 1980-1983, Marine Mining, in press.

Lister, C. R. B., On the penetration of water into hot rock, Geophys. J. Roy. Astr. Soc., 39, 465-509, 1974.

Lister, C. R. B., On the intermittency and crystallization mechanisms of sub-seafloor magma chambers, Geophys. J. Roy. Astr. Soc., 73, 351-366, 1983.

Lowell, R. P., Circulation in fractures, hot springs, and convective heat transport on mid-ocean ridge crests, Geophys. J. Roy. Astr. Soc., 40, 351-365, 1975.

Lowell, R. P., Comments on "Theory of heat extraction from fractured hot dry rock" by A. C. Gringarten, P. A. Witherspoon, and Yuzo Ohnishi, J. Geophys. Res., 81, 359, 1976.

Macdonald, K. C., Mid-ocean ridges: Five scale tectonic, volcanic and hydrothermal processes within the plate boundary zone, Ann. Rev., Earth Planet. Sci., 10, 155-190, 1982.

Macdonald, K. C., Crustal processes at spreading centers, Rev. Geophys. Space Phys., 21, No. 6, 1441-1454, 1983a.

Macdonald, K. C., K. Becker, F. N. Spiess, and R. D. Ballard, Hydrothermal heat flux of the "black smoker" vents on the East Pacific Rise, Earth Planet. Science Lett., 48, 1-7, 1980.

Macdonald, K. C., A geophysical comparison between fast- and slow-spreading centers: Constraints on magma chamber formation and hydrothermal activity, in Hydrothermal Processes at Seafloor Spreading Centers, edited by P. A. Rona, K. Bostrom, L. Laubier, and K. Smith, NATO Marine Science Series IV, v. 12, Plenum Press, NY, 1984 (in press).

Malahoff, A., R. W. Embley, D. S. Cronan, and R. Skirrow, The geological setting and chemistry of hydrothermal sulphides and associated deposits from the Galapagos Rift at 86^{o}W, Marine Mining, 4, 123-137, 1983.

Miller, A. R., C. D. Densmore, E. T. Degens, F. C. Hathaway, F. T. Manheim, P. F. McFarlen, H. Pocklington, and A. Jokela, Hot brines and recent iron deposits in deeps of the Red Sea, Geochim. Cosmochim. Acta, 30, 341-359, 1966.

Mining Magazine, Etibank's role in the Turkish mining industry, 140, no. 6, 501, 1979a.

Mining Magazine, Oman copper project underway, 141, no. 9, 169, 1979b.

Mosier, D. L., D. A. Singer, and B. B. Salem, Geologic and grade-tonnage information on volcanic-hosted copper-zinc-lead massive sulphide deposits, U.S. Geol. Surv. Open-File Rep., 83-89, 77 pp., 1983.

Nawab, Z. A., Red Sea mining: A new era, in Marine Science of the Northwest Indian Ocean and Adjacent Waters, edited by M. V. Angel, pp. 813-822, Proceedings of the Mabahiss John Murray Internat. Symposium, Egypt, 3-6 September 1983, Deep-Sea Research, Part A. Oceanographic Research Papers, 31, Nos. 6-8A.

Nelson, N. E., Hidden Creek mine, in Symposium on Structural Geology of Canadian Ore Deposits, Montreal, Canadian Institute of Mining and Metallurgy, 1, 125-128, 1948.

Pallister, J. S., and C. A. Hopson, Samail ophiolite plutonic suite: field relations, phase variation, cryptic variation and layering, and a model of a spreading ridge magma chamber, J. Geophys. Res., 86, 2593-2644, 1981.

Parmentier, E. M., and E. T. C. Spooner, A theoretical study of hydrothermal convection and the origin of the ophiolitic sulphide ore deposits of Cyprus, Earth Planet. Sci. Lett., 40, 33-44, 1978.

Ribando, R. J., K. E. Torrance, and D. L. Turcotte, Numerical models for hydrothermal circulation in the oceanic crust, J. Geophys. Res., 81, 3007-3012, 1976.

RISE Project Group (F. N. Spiess, K. C. Macdonald, T. Atwater, R. D. Ballard, A. Carranza, D. Cordoba, C. Cox, V. M. Diaz Garcia, J. Francheteau, J. Guerrero, J. Hawkins, R. Hayman, R. Hessler, T. Juteau, M. Kastner, R. Larson, B. Luyendyk, J. D. Macdougall, S. Miller, W. Normark, J. Orcutt, and C. Rangin), East Pacific Rise: hot springs and geophysical experiments, Science, 207, 1421-1444, 1980.

Rona, P. A., Exploration for hydrothermal mineral deposits at seafloor spreading centers, Marine Mining, 4, 7-38, 1983.

Rona, P. A., Hydrothermal mineralization at seafloor spreading centers, Earth-Science Reviews, 20, 1-104, 1984.

Rona, P. A., and R. P. Lowell, eds., Seafloor Spreading Centers: Hydrothermal Systems, Benchmark Papers in Geology, v. 56, Hutchinson and Ross, Stromberg, PA, 424 p., 1980.

Rona, P. A., K. Bostrom, L. Laubier, and K. L. Smith, Jr. (editors), Hydrothermal Processes at Seafloor Spreading Centers, NATO Conference Series, IV, Marine Sciences, 12, Plenum Press, New York, 1-796, 1983.

Rona, P. A., G. Thompson, M. J. Mottl, J. A. Karson, W. J. Jenkins, D. Graham, M. Mallette, K. Von Damm, and J. M. Edmond, Hydrothermal activity at the Trans-Atlantic Geotraverse hydrothermal field, mid-Atlantic ridge crest at 26^{o}N, J. Geophys. Res., 89, 11365-11377, 1984.

Saudi-Sudanese Red Sea Joint Commission, 1982. The Red Sea (brochure), 4 p.

Schott, W. (editor), Die fahrten des forschungsschiffes, VALDIVIA, 1971-1978, Geowissenschaftliche ergebnisse, Geol. Jb. D38, 204 p., 1980.

Searle, D. L., Mode of occurrence of the cupriferous pyrite deposits of Cyprus, London, Institute of Mining and Metallurgy, Transaction, sec. B, 81, B189-B197, 1972.

Sleep, N. H., and T. J. Wolery, Thermal and mechanical constraints on venting of hydrothermal fluids at mid-ocean ridges, J. Geophys. Res., 83, 5913-5922, 1978.

Solomon, M., "Volcanic" massive sulphide deposits and their host rocks--a review and explanation, in Handbook of Strata-Bound and Strata-form Ore Deposits, v. 6, ed. by K. H. Wolf, Elsevier, Amsterdam, p. 21-54, 1976.

Solomon, M., and J. L. Walshe, The formation of massive sulphide deposits on the seafloor, Economic Geology, 74, 797-813, 1979.

Spooner, E. T. C., Hydrodynamic model for the origin of ophiolitic cupiferous pyrite ore deposits of Cyprus, in Volcanic Processes in Ore Genesis (Institution of Mining and Metallurgy and Geological Society, London), p. 58-71, 1977.

Straus, J. M., and G. Schubert, Thermal convection of water in porous medium: effects of temperature and pressure dependent thermal properties, J. Geophys. Res., 82, 325-333, 1977.

Strens, M. R., and J. R. Cann, A model of hydrothermal circulation in fault zones at mid-ocean ridge crests, Geophys. J. R. Astr. Soc., 71, 225-240, 1982.

Swanberg, C. A., The Mesa geothermal anomaly, Imperial Valley, California: A comparison and evaluation of results obtained from surface geophysics and deep drilling, in Proc. 2nd U. N. Symposium and Use of Geothermal Resources, pp. 1217-1229, San Francisco, v. 2, 1975.

Tivey, M. K., and J. R. Delaney, Sulfide deposits from the Endeavour Segment of the Juan de Fuca Ridge, Marine Mining, in press.

Todd, S. G., D. W. Keith, D. J. Schissel, L. L. LeRoy, E. L. Mann, and I. H. Irvine, The J-M platinum-palladium reef of the Stillwater Complex, Montana: I. Stratigraphy and petrology, Econ. Geol., 77, 1454-1480, 1982.

Upadhyay, H. D., and D. F. Strong, Geologic setting of the Betts Cove copper deposits, Newfoundland: an example of ophiolite mineralization, Economic Geology, 68, no. 2, 161-167, 1973.

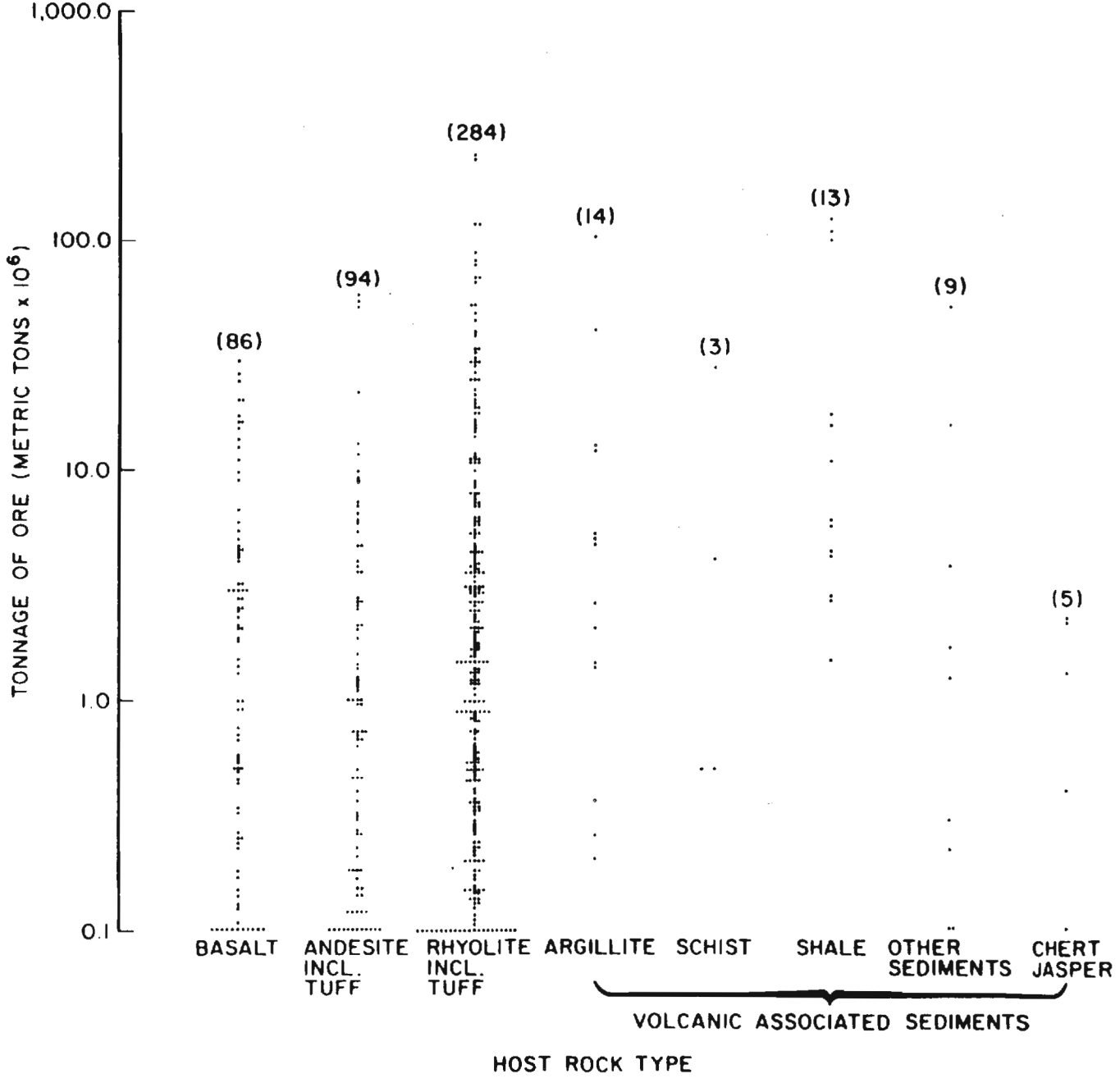
Vokes, F. M., Mines in south and central Norway, Norges Geologiske Undersokelse, no. 212, 1-73, 1960.

Wager, L. R., and G. M. Brown, Layered Igneous Rocks, Oliver and Boyd, Edinburgh, 588 p., 1968.

FIGURE CAPTIONS

Figure 1. Plot of size of 508 massive sulphide ore deposits on land versus type of volcanic host rock immediately underlying each deposit based on a data compilation by Mosier et al. (1983). The size of known massive sulphide deposits hosted in basalt including basalt generated at seafloor spreading centers (Table 1) ranges up to 30 x 10^6 metric tons. The size of known massive sulphide deposits hosted in andesite and rhyolite ranges up to 242 x 10^6 metric tons. The number in parentheses above each column of points is the number of massive sulphide deposits plotted.

Figure 2. Plot of total mass discharge per km^2 of magma contact area, at a temperature greater than or equal to 350°C, as a function of permeability, K, in the contact zone where the hydrothermal fluid is heated.


1,000.0
100.0
10.0
1.0
0.1
TONNAGE OF ORE (METRIC TONS x 10^6)
(86)
(94)
(284)
(14)
(3)
(13)
(9)
(5)
BASALT
ANDESITE INCL. TUFF
RHYOLITE INCL. TUFF
ARGILLITE
SCHIST
SHALE
OTHER SEDIMENTS
CHERT JASPER
VOLCANIC ASSOCIATED SEDIMENTS
HOST ROCK TYPE

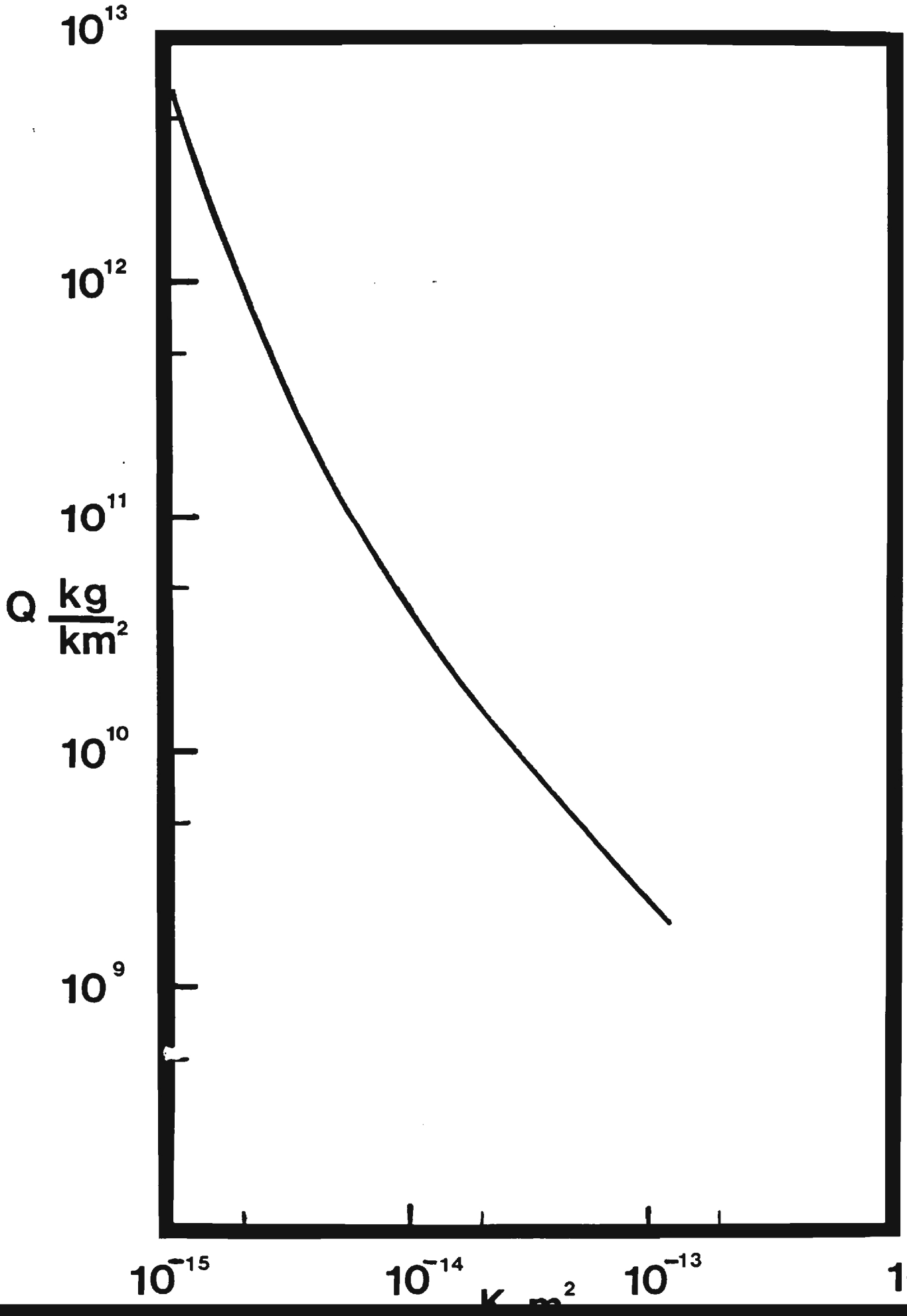


FIGURE 2.

Au	Age	Location	Position		Comments	Reference
			Latitude	Longitude		
0.000005	Recent	Red Sea	21°23'N	38°03'E	Reliable assessment based on three-dimensional sampling	Bäcker, 1979; Schott 1980; Saudi-Sudanese Red Sea Joint Commission, 1982; Nawab, 1984
----	Recent	Pacific Ocean	20°50'N	109°05'W	Tentative assessment based on surface sampling	Hekinian et al., 1980; RISE, 1980; Rona, 1984
----	Recent	Pacific Ocean	0°45'N	85°50'W	Tentative assessment based on surface sampling	Malahoff et al., 19
----	Recent	Pacific Ocean	44°40'N	130°20'W	Tentative assessment based on surface sampling	Koski et al., 1984; Koski et al., in press
0.000002	Recent	Pacific Ocean	47°56.7'N	129°05.8'W	Tentative assessment based on surface sampling	Tivey and Delaney, in press

			Position			
Au	Age	Location	Latitude	Longitude	Comment	Reference
0001	Cretaceous	Turkey	38°22'N	39°40'E	Possible ophiolite	Griffitts et al., 1972; Mining Magazine, 1979a; Mosier et al., 1983
0	Silurian	Spain	42°53'N	08°33'W	Possible ophiolite	Badham and Williams, 1981; Mosier et al., 1983
00002	Jurassic	Canada	55°26'N	129°49'W	Uncertain origin	Nelson, 1948; Mosier et al., 1983
0	Cretaceous	Oman	24°00'N	55°00'E	Semail ophiolite	Alabaster et al., 1980; Mining Magazine, 1979b; Mosier et al., 1983
.00003	Ordovician	Norway	63°08'N	09°45'E	Ophiolite	Bugge, 1978; Franklin et al., 1981; Frietsch et al., 1979; Grenne et al., 1980; Vokes, 1960, Waltham, 1968; Mosier et al., 1983
0.003	Cretaceous	Cyprus	35°05'N	32°50'W	Troodos ophiolite	Hutchinson, 1965; Searle, 1972; Mosier et al., 1983
0	Cretaceous	Turkey	38°00'N	42°00'E	Possible ophiolite	Mosier et al., 1983
.00008	Ordovician	Canada	49°52'N	55°38'W	Newfoundland ophiolite	Donoghue et al., 1959; Douglas et al., 1940; Upadhyay and Strong, 1973; Mosier et al., 1983

TABLE 3. Water-Rock Contact Area as a Function of Time of Formation.

BASED ON: $T_w(h,t) = 350^oC$, $T_{wo} = 100^oC$, $T_{Ro} = 700^oC$,
$\lambda = 2.5$ w/m-^oC, $a = 10^{-6}$ m^2/sec, $s = 8 \times 10^3$ j/kg-^oC

t (years)	m = Qt (kg)	Q = qℓ (kg/sec)	A = hℓ (km^2)
10^2	10^{13}	3×10^3	200
10^3	10^{13}	3×10^2	63
10^4	10^{13}	30	20
10^5	10^{13}	3	6.3